8.66% APR is based on a Prime Rate of 8.25% as of August 1, 2006.

CHAPTER

1 Operations with Real Numbers

While on a trip to Canada, Tonya heard on the radio that the temperature today will be 20° Celsius. Will she need her winter coat? What will the temperature be in degrees Fahrenheit? We can determine what 20° Celsius is in degrees Fahrenheit by the following expression.

$$F = \frac{9}{5} \cdot 20 + 32$$

1–1 ■ Operations with fractions

Fractions

In day-to-day living, the numbers we use most often are the whole numbers,

$$0, 1, 2, 3, 4, \text{ and so on,}$$

for counting and the fractions, such as

$$\frac{1}{2}, \frac{3}{4}, \frac{9}{10}, \text{ and so on.}$$

In a fraction, the top number is called the **numerator** and the bottom number is called the **denominator.**

$$\frac{9 \longleftarrow \text{Numerator}}{10 \longleftarrow \text{Denominator}}$$

There are two types of fractions:

1. **Proper fractions** where the numerator is less than the denominator; for example, $\frac{9}{10}$.
2. **Improper fractions** where the numerator is greater than or equal to the denominator; for example, $\frac{10}{9}$ or $\frac{9}{9}$.

Prime numbers and factorization

Any whole number can be stated as a **product** of two or more whole numbers, called **factors** of the number. For example,

Product	Factors	
12	$= 2 \cdot 6$	2 and 6 are factors
12	$= 1 \cdot 12$	1 and 12 are factors
12	$= 4 \cdot 3$	4 and 3 are factors
12	$= 2 \cdot 2 \cdot 3$	2, 2, and 3 are factors

To factor a whole number is to write the number as a product of factors. In future work, it will be necessary to factor whole numbers such that the factors are **prime numbers.**

Prime numbers

A prime number is any whole number greater than 1 whose only factors are the number itself and 1.

The first ten prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

Thus when we factored 12 as $12 = 2 \cdot 2 \cdot 3$, the number was stated as a product of prime factors.

■ *Example 1–1 A*

Write each number as a product of prime factors.

1. 36

$2 \cdot 18$ Divide $36 \div 2 = 18$

$2 \cdot 2 \cdot 9$ Divide $18 \div 2 = 9$

$2 \cdot 2 \cdot 3 \cdot 3$ Divide $9 \div 3 = 3$

Thus $36 = 2 \cdot 2 \cdot 3 \cdot 3$

Note This could have been done in the following way.

2 |36 Divide $36 \div 2 = 18$
2 |18 Divide $18 \div 2 = 9$
3 |9 Divide $9 \div 3 = 3$
3

We successively divide by prime numbers, starting with 2 if possible, until the quotient is a prime number.

2. 54

2 |54 Divide $54 \div 2 = 27$
3 |27 Divide $27 \div 3 = 9$
3 |9 Divide $9 \div 3 = 3$
3

Thus $54 = 2 \cdot 3 \cdot 3 \cdot 3$. ■

Reducing fractions to lowest terms

A fraction is **reduced to lowest terms** when the only factor common to the numerator and the denominator is 1.

To reduce a fraction to lowest terms

1. Write the numerator and the denominator as a product of prime factors.
2. Divide the numerator and the denominator by all common factors.

Example 1–1 B

1. $\frac{14}{21} = \frac{2 \cdot 7}{3 \cdot 7}$ Write as a product of prime factors

$= \frac{2}{3}$ Divide numerator and denominator by common factor 7

$\frac{2}{3}$ is the answer since 2 and 3 have only 1 as a common factor.

2. $\frac{45}{60} = \frac{3 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5}$ Write as a product of prime factors

$= \frac{3}{2 \cdot 2}$ Divide numerator and denominator by $3 \cdot 5$

$= \frac{3}{4}$ Multiply in denominator

▶ *Quick check* Reduce $\frac{25}{45}$ to lowest terms.

Products and quotients of fractions

To multiply two or more fractions, we use the following procedure.

To multiply fractions

1. Write the numerator and the denominator as an indicated product (do not multiply).
2. Reduce the resulting fraction to lowest terms.

Example 1–1 C

Multiply the following fractions and reduce to lowest terms.

1. $\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7}$ Multiply numerators
Multiply denominators

$= \frac{10}{21}$ Perform multiplications

2. $\frac{5}{6} \cdot \frac{3}{4} = \frac{5 \cdot 3}{6 \cdot 4}$ Multiply numerators
Multiply denominators

$= \frac{5 \cdot 3}{2 \cdot 3 \cdot 2 \cdot 2}$ Factor $6 = 2 \cdot 3$, $4 = 2 \cdot 2$

$= \frac{5}{2 \cdot 2 \cdot 2}$ Divide numerator and denominator by 3

$= \frac{5}{8}$ Multiply in the denominator

Suppose we multiply the fractions

$$\frac{5}{6} \cdot \frac{6}{5} = \frac{5 \cdot 6}{6 \cdot 5} = \frac{30}{30} = 1$$

When the product of two numbers is 1, we call each number the **reciprocal** of the other number. Thus

$$\frac{5}{6} \text{ and } \frac{6}{5} \text{ are reciprocals,}$$
$$\frac{2}{7} \text{ and } \frac{7}{2} \text{ are reciprocals,}$$
$$\frac{14}{13} \text{ and } \frac{13}{14} \text{ are reciprocals.}$$

We can see that the reciprocal of any fraction is obtained by interchanging the numerator and the denominator. The reciprocal of a fraction is used to divide fractions.

To divide two fractions

1. Multiply the first fraction by the **reciprocal** of the second fraction.
2. Reduce the resulting product to lowest terms.

Example 1–1 D

Divide the following fractions and reduce to lowest terms.

1. $\frac{7}{8} \div \frac{6}{7} = \frac{7}{8} \cdot \frac{7}{6}$ Multiply by the reciprocal of $\frac{6}{7}$

$= \frac{7 \cdot 7}{8 \cdot 6}$ Multiply numerators / Multiply denominators

$= \frac{49}{48}$

Note The improper fraction $\frac{49}{48}$ can be written as the **mixed number** $1\frac{1}{48}$, which is the sum of a whole number and a proper fraction. This is obtained by dividing the numerator by the denominator.

$$48\overline{)49} \quad 1 \text{ (Quotient)}; \quad 49 - 48 = 1 \text{ Remainder}$$

$$= 1\frac{1}{48}$$

Quotient
Remainder
Original denominator
Remainder

2. $\dfrac{\frac{4}{5}}{\frac{3}{7}} = \frac{4}{5} \div \frac{3}{7} = \frac{4}{5} \cdot \frac{7}{3}$ Multiply by the reciprocal of $\frac{3}{7}$

$= \frac{4 \cdot 7}{5 \cdot 3}$ Multiply numerators / Multiply denominators

$= \frac{28}{15}$ or $1\frac{13}{15}$ Perform indicated operations

The improper fraction answer is usually the one preferred in algebra. The mixed number form is usually preferred in an application problem.

3. $3\frac{1}{4} \div 5\frac{2}{3}$

We change the mixed numbers to improper fractions.

Denominator times whole number

Plus numerator

$$3\frac{1}{4} = \frac{(3 \cdot 4) + 1}{4} = \frac{12 + 1}{4} = \frac{13}{4}$$

Mixed number form

Original denominator

Improper fraction form

$$5\frac{2}{3} = \frac{(3 \cdot 5) + 2}{3} = \frac{15 + 2}{3} = \frac{17}{3}$$

We now divide as indicated.

$$3\frac{1}{4} \div 5\frac{2}{3} = \frac{13}{4} \div \frac{17}{3}$$

$= \frac{13}{4} \cdot \frac{3}{17}$ Multiply by the reciprocal of $\frac{17}{3}$

$= \frac{13 \cdot 3}{4 \cdot 17}$ Multiply numerators / Multiply denominators

$= \frac{39}{68}$ Perform indicated operations

▶ ***Quick check*** Divide $\dfrac{\frac{5}{6}}{\frac{5}{8}}$ and reduce to lowest terms.

4. The area of a rectangle is found by multiplying the length of the rectangle by the width of the rectangle. Find the area of a rectangle that is $2\frac{1}{2}$ feet long and $1\frac{5}{6}$ feet wide.

Area $= 2\frac{1}{2} \cdot 1\frac{5}{6}$ Multiply the given dimensions

$= \frac{5}{2} \cdot \frac{11}{6}$ Change mixed numbers to improper fractions

$= \frac{5 \cdot 11}{2 \cdot 6}$ Multiply numerators / Multiply denominators

$= \frac{55}{12}$ or $4\frac{7}{12}$ Perform indicated operations

The area of the rectangle is $4\frac{7}{12}$ square feet. ■

Addition and subtraction of fractions

To add or subtract fractions, the fractions must have a *common* (same) *denominator.*

To add or subtract fractions with common denominators

1. Add or subtract the numerators.
2. Place the sum or difference over the common denominator.
3. Reduce the resulting fraction to lowest terms.

Example 1–1 E

Add or subtract the following fractions as indicated. Reduce to lowest terms.

1. $\frac{3}{8} + \frac{1}{8} = \frac{3+1}{8}$ Add numerators

$= \frac{4}{8}$ Combine in numerator

$= \frac{1}{2}$ Reduce to lowest terms

2. $\frac{7}{16} - \frac{5}{16} = \frac{7-5}{16}$ Subtract numerators

$= \frac{2}{16}$ Combine in numerator

$= \frac{1}{8}$ Reduce to lowest terms

When the fractions have different denominators, we must rewrite all of the fractions with a new common denominator. Many numbers can satisfy the condition for any set of denominators, but we want the *least* of these numbers, called the **least common denominator** (denoted by LCD). For example, 24 is the least common denominator of the fractions

$$\frac{7}{8} \text{ and } \frac{5}{6}$$

since it is the least (smallest) number that can be divided by 6 and 8 exactly. The procedure for finding the LCD is outlined next.

To find the least common denominator (LCD)

1. Express each denominator as a product of prime factors.
2. List all the *different* prime factors.
3. Write each prime factor the *greatest* number of times it appears in any of the prime factorizations in step 1.
4. The least common denominator is the product of all factors from step 3.

■ ***Example 1–1 F***

Find the least common denominator (LCD) of the fractions with the following denominators.

1. 24 and 18

a. Write 24 and 18 as products of prime factors.

$24 = 2 \cdot 2 \cdot 2 \cdot 3$

$18 = 2 \cdot 3 \cdot 3$

b. The different prime factors are 2 and 3.

c. 2 is a factor three times in 24 and 3 is a factor two times in 18 (the greatest number of times).

d. The LCD is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$.

2. 6, 8, and 14

a. $6 = 2 \cdot 3$

$8 = 2 \cdot 2 \cdot 2$

$14 = 2 \cdot 7$

b. The different prime factors are 2, 3, and 7.

c. 2 is a factor three times in 8, 3 is a factor once in 6, and 7 is a factor once in 14.

d. The LCD is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$.

▶ ***Quick check*** Find the LCD for fractions with denominators of 6, 9, and 12. ■

Building fractions

To write the fraction $\frac{5}{6}$ as an equivalent fraction with new denominator 24, we find the number that is multiplied by 6 to get 24. Since

$$6 \cdot 4 = 24$$

we use the factor 4. Now multiply the given fraction $\frac{5}{6}$ by the fraction $\frac{4}{4}$. The fraction $\frac{4}{4}$ is equal to 1 and is called a unit fraction. Multiplying by the unit fraction $\frac{4}{4}$ will not change the value of $\frac{5}{6}$, only its form. Thus

$$\frac{5}{6} = \frac{5}{6} \cdot \underbrace{\frac{4}{4}}_{\text{Multiplication by 1}} = \frac{5 \cdot 4}{6 \cdot 4} = \frac{20}{24}$$

We use the following procedure to write equivalent fractions.

To find equivalent fractions

1. Divide the original denominator into the new denominator.
2. Multiply the numerator and the denominator of the given fraction by the number obtained in step 1.

Example 1–1 G

Write equivalent fractions having the new denominator.

1. $\frac{3}{5} = \frac{?}{30}$

Since $30 \div 5 = 6$, multiply $\frac{3}{5}$ by $\frac{6}{6}$.

$$\frac{3}{5} = \frac{3}{5} \cdot \frac{6}{6} \quad \text{Multiply by } \frac{6}{6}$$
$$= \frac{3 \cdot 6}{5 \cdot 6} \quad \text{Multiply numerators} \quad \text{Multiply denominators}$$
$$= \frac{18}{30}$$

2. $\frac{7}{9} = \frac{?}{72}$

Since $72 \div 9 = 8$, multiply $\frac{7}{9}$ by $\frac{8}{8}$.

$$\frac{7}{9} = \frac{7}{9} \cdot \frac{8}{8} \quad \text{Multiply by } \frac{8}{8}$$
$$= \frac{7 \cdot 8}{9 \cdot 8} \quad \text{Multiply numerators} \quad \text{Multiply denominators}$$
$$= \frac{56}{72}$$

To add or subtract fractions having different denominators, we use the following procedure.

To add or subtract fractions having different denominators

1. Find the LCD of the fractions.
2. Write each fraction as an equivalent fraction with the LCD as the new denominator.
3. Perform the addition or subtraction as before.
4. Reduce the resulting fraction to lowest terms.

Example 1–1 H

Add or subtract the following fractions as indicated. Reduce the resulting fraction to lowest terms.

1. $\frac{7}{8} + \frac{5}{6}$

a. The LCD of the fractions is 24.

b. Since $24 \div 8 = 3$, then

$$\frac{7}{8} = \frac{7}{8} \cdot \frac{3}{3} = \frac{7 \cdot 3}{8 \cdot 3} = \frac{21}{24} \quad \text{Multiply by } \frac{3}{3}$$

and since $24 \div 6 = 4$, then

$$\frac{5}{6} = \frac{5}{6} \cdot \frac{4}{4} = \frac{5 \cdot 4}{6 \cdot 4} = \frac{20}{24} \quad \text{Multiply by } \frac{4}{4}$$

c. $\frac{7}{8} + \frac{5}{6} = \frac{21}{24} + \frac{20}{24}$ Add fractions with LCD

$= \frac{21 + 20}{24}$ Add numerators

$= \frac{41}{24}$ or $1\frac{17}{24}$

2. $\frac{7}{8} - \frac{1}{3}$

a. The LCD of the fractions is 24.

b. Since $24 \div 8 = 3$, then

$\frac{7}{8} = \frac{7}{8} \cdot \frac{3}{3} = \frac{21}{24}$ Multiply by $\frac{3}{3}$

Since $24 \div 3 = 8$, then

$\frac{1}{3} = \frac{1}{3} \cdot \frac{8}{8} = \frac{8}{24}$ Multiply by $\frac{8}{8}$

c. $\frac{7}{8} - \frac{1}{3} = \frac{21}{24} - \frac{8}{24}$ Subtract fractions with LCD

$= \frac{21 - 8}{24} = \frac{13}{24}$ Subtract numerators

3. $3\frac{7}{8} - 2\frac{3}{4}$

Change each of the mixed numbers to an improper fraction.

$3\frac{7}{8} = \frac{(8 \cdot 3) + 7}{8} = \frac{31}{8}$; $\quad 2\frac{3}{4} = \frac{(4 \cdot 2) + 3}{4} = \frac{11}{4}$

a. The LCD of the fractions is 8.

b. $\frac{31}{8}$ already has the LCD in its denominator.

Since $8 \div 4 = 2$, then

$\frac{11}{4} = \frac{11 \cdot 2}{4 \cdot 2} = \frac{22}{8}$ Multiply by $\frac{2}{2}$

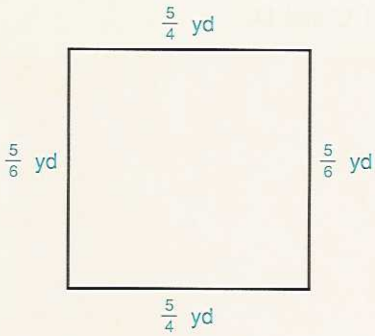
c. $3\frac{7}{8} - 2\frac{3}{4} = \frac{31}{8} - \frac{11}{4}$ Subtract improper fractions

$= \frac{31}{8} - \frac{22}{8}$ Subtract fractions with LCD

$= \frac{31 - 22}{8}$ Subtract numerators

$= \frac{9}{8}$ or $1\frac{1}{8}$

4. The perimeter (distance around) of a rectangle is found by *adding* the lengths of the four sides of the rectangle. Find the perimeter of a rectangle that is $1\frac{1}{4}$ yards long and $\frac{5}{6}$ of a yard wide.



$$
\begin{aligned}
\text{Perimeter} &= \frac{5}{6} + 1\frac{1}{4} + \frac{5}{6} + 1\frac{1}{4} && \text{Add all the sides}\\
&= \frac{5}{6} + \frac{5}{4} + \frac{5}{6} + \frac{5}{4} && \text{Change to improper fractions}\\
&= \frac{10}{12} + \frac{15}{12} + \frac{10}{12} + \frac{15}{12} && \text{LCD is 12}\\
&= \frac{10 + 15 + 10 + 15}{12} && \text{Add numerators}\\
&= \frac{50}{12} = \frac{25}{6} && \text{Reduce}\\
&\text{or } 4\frac{1}{6} && \text{Mixed number form}
\end{aligned}
$$

The perimeter of the rectangle is $4\frac{1}{6}$ yards.

▶ ***Quick check*** $4\frac{1}{2} - 2\frac{3}{4}$ ■

Mastery points

Can you

- Reduce a fraction to lowest terms?
- Multiply and divide fractions?
- Find the least common denominator (LCD) of two or more fractions?
- Add and subtract fractions?

Exercise 1–1

Reduce the following fractions to lowest terms. See example 1–1 B.

Example $\frac{25}{45}$

Solution

$$
\begin{aligned}
\frac{25}{45} &= \frac{5 \cdot 5}{3 \cdot 3 \cdot 5} && \text{Factor numerator}\\
& && \text{Factor denominator}\\
&= \frac{5}{3 \cdot 3} && \text{Divide numerator and denominator by 5}\\
&= \frac{5}{9} && \text{Multiply in denominator}
\end{aligned}
$$

1. $\frac{4}{8}$ **2.** $\frac{3}{9}$ **3.** $\frac{10}{12}$ **4.** $\frac{8}{14}$ **5.** $\frac{16}{18}$ **6.** $\frac{14}{21}$

$\boxed{7.}$ $\frac{28}{36}$ **8.** $\frac{50}{75}$ **9.** $\frac{64}{32}$ **10.** $\frac{96}{48}$ **11.** $\frac{100}{85}$ **12.** $\frac{120}{84}$

Multiply or divide the fractions as indicated. Reduce to lowest terms. See examples 1–1 C and D.

Example $\dfrac{\frac{5}{6}}{\frac{5}{8}}$

Solution $\dfrac{\frac{5}{6}}{\frac{5}{8}} = \dfrac{5}{6} \div \dfrac{5}{8} = \dfrac{5}{6} \cdot \dfrac{8}{5}$ Multiply by the reciprocal of $\frac{5}{8}$

$= \dfrac{5 \cdot 8}{6 \cdot 5}$ Multiply numerators / Multiply denominators

$= \dfrac{8}{6}$ Reduce by common factor 5

$= \dfrac{2 \cdot 2 \cdot 2}{2 \cdot 3}$ Write numerator and denominator as the product of prime factors

$= \dfrac{2 \cdot 2}{3}$ Reduce by common factor 2

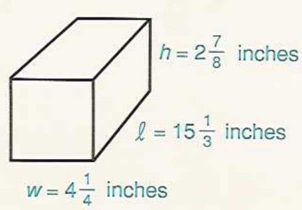
$= \dfrac{4}{3}$ or $1\dfrac{1}{3}$ Multiply in denominator

13. $\dfrac{5}{6} \cdot \dfrac{3}{5}$
14. $\dfrac{2}{3} \cdot \dfrac{5}{6}$
15. $\dfrac{7}{8} \cdot \dfrac{7}{12}$
16. $\dfrac{7}{5} \cdot \dfrac{3}{2}$
17. $\dfrac{7}{9} \cdot \dfrac{3}{4}$
18. $\dfrac{3}{4} \cdot 6$
19. $\dfrac{3}{7} \div \dfrac{4}{5}$
20. $\dfrac{12}{25} \div \dfrac{8}{15}$
21. $\dfrac{6}{7} \div 3$
22. $4 \div \dfrac{3}{8}$
23. $\dfrac{15}{17} \div \dfrac{3}{5}$
24. $4 \div \dfrac{7}{2}$
25. $17 \div 2\dfrac{1}{3}$
26. $12 \cdot 1\dfrac{5}{6}$
27. $7\dfrac{1}{3} \cdot 2\dfrac{4}{7}$
28. $1\dfrac{1}{5} \cdot 2\dfrac{1}{2}$
29. $4\dfrac{4}{5} \cdot 2\dfrac{1}{2}$
30. $7\dfrac{1}{2} \div 5\dfrac{1}{4}$
31. $\dfrac{8}{\frac{2}{3}}$
32. $\dfrac{17}{\frac{3}{4}}$
33. $\dfrac{\frac{7}{8}}{\frac{4}{3}}$
34. $\dfrac{\frac{15}{64}}{\frac{45}{8}}$
35. $\dfrac{4}{5} \cdot \dfrac{2}{3} \cdot \dfrac{3}{8}$
36. $\dfrac{9}{8} \cdot \dfrac{2}{3} \cdot \dfrac{3}{8}$
37. $\dfrac{8}{3} \cdot \dfrac{4}{7}$
38. $\dfrac{8}{3} \div \dfrac{15}{14}$
39. $\dfrac{8}{3} \cdot \dfrac{15}{14}$

See example 1–1 D–4.

40. What is the total length of 25 pieces of steel, each $5\frac{1}{2}$ inches long?

41. The volume of a rectangular block is found by multiplying the length times the width times the height.



a. What is the volume in cubic inches of a rectangular block of wood $15\frac{1}{3}$ inches long, $4\frac{1}{4}$ inches wide, and $2\frac{7}{8}$ inches high?

b. What is the volume in cubic inches of a block of steel $8\frac{1}{2}$ inches long, $2\frac{1}{8}$ inches wide, and $1\frac{3}{4}$ inches high?

$\boxed{\textbf{42.}}$ A wire $61\frac{1}{2}$ inches long is divided into 14 equal parts. What is the length of each part?

Find the LCD of the fractions with the following groups of denominators. See example 1–1 F.

Example 6, 9, and 12

Solution
1. State 6, 9, and 12 as a product of prime factors.
$$6 = 2 \cdot 3$$
$$9 = 3 \cdot 3$$
$$12 = 2 \cdot 2 \cdot 3$$
2. The different prime factors are 2 and 3.
3. 2 is a factor twice in 12 and 3 is a factor twice in 9.
4. The LCD is $2 \cdot 2 \cdot 3 \cdot 3 = 36$.

43. 3, 8, 10 **44.** 9, 15, 21 $\boxed{\textbf{45.}}$ 6, 14, 18 **46.** 5, 10, 12

47. 16, 24, 36 **48.** 12, 16, 24 **49.** 5, 7, 11 **50.** 10, 20, 30

51. 68, 9, 12 **52.** 10, 14, 18 **53.** 10, 15, 20 **54.** 10, 15, 24

Add or subtract the following fractions as indicated. Reduce to lowest terms. See examples 1–1 E and H.

Example $4\frac{1}{2} - 2\frac{3}{4}$

Solution We first change the mixed numbers to improper fractions.

$$4\frac{1}{2} = \frac{(4 \cdot 2) + 1}{2} = \frac{9}{2}; \quad 2\frac{3}{4} = \frac{(4 \cdot 2) + 3}{4} = \frac{11}{4}$$

$$4\frac{1}{2} - 2\frac{3}{4} = \frac{9}{2} - \frac{11}{4}$$ Replace mixed numbers with improper fractions

The LCD is 4.

$$= \frac{18}{4} - \frac{11}{4}$$ Write $\frac{9}{2}$ as $\frac{18}{4}$

$$= \frac{18 - 11}{4}$$ Subtract numerators

$$= \frac{7}{4} \text{ or } 1\frac{3}{4}$$

55. $\frac{1}{3} + \frac{1}{3}$ **56.** $\frac{2}{5} + \frac{3}{10}$ **57.** $\frac{1}{3} + \frac{1}{4}$ **58.** $\frac{5}{6} - \frac{1}{6}$

59. $\frac{4}{5} - \frac{2}{10}$ **60.** $\frac{5}{6} - \frac{3}{8}$ $\boxed{\textbf{61.}}$ $1 + \frac{5}{8}$ **62.** $3 + \frac{5}{6}$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63. $4 - \frac{3}{5}$

64. $\frac{2}{3} + \frac{3}{4}$

65. $\frac{3}{5} + \frac{7}{15}$

66. $\frac{5}{6} - \frac{1}{3}$

67. $\frac{3}{8} - \frac{1}{12}$

68. $\frac{7}{24} - \frac{3}{16}$

69. $\frac{7}{54} + \frac{19}{45}$

70. $\frac{1}{2} + \frac{1}{5} + \frac{1}{10}$

71. $\frac{7}{15} + \frac{5}{6} - \frac{3}{4}$

72. $\frac{9}{16} + \frac{5}{18} - \frac{2}{15}$

73. $8\frac{3}{16} - 4\frac{5}{8}$

74. $7\frac{1}{2} + 2\frac{3}{4}$

75. $\frac{2}{7} + \frac{2}{3} + \frac{5}{7}$

See example 1–1 H–4.

76. Jane owed Joan some money. If she paid Joan $\frac{1}{4}$ of the debt on June 15, $\frac{1}{3}$ of the original debt on July 1, and $\frac{3}{8}$ of the original debt on August 10, how much of her debt had Jane paid by August 10?

77. A flower garden in the form of a rectangle has two sides that are $24\frac{1}{2}$ feet long and two sides that are $18\frac{3}{4}$ feet long. Find the perimeter (total distance around) of the rectangle.

78. On a given day, Mrs. Jones purchased $\frac{5}{6}$ yard of one material, $\frac{3}{4}$ yard of another material, and $\frac{2}{3}$ yard of a third material. How many yards of material did she purchase altogether?

79. Butcher John has $32\frac{1}{4}$ pounds of pork chops. If he sells $21\frac{1}{3}$ pounds of the pork chops on a given day, how many pounds of pork chops does he have left?

80. A machinist has a piece of steel stock that weighs $12\frac{7}{8}$ ounces. If he cuts off $5\frac{1}{5}$ ounces, how many ounces does he have left?

1–2 ■ Operations with decimals and percents

In section 1–1, we studied fractions. A **decimal number** is a special fraction with a denominator that is 10, 100, 1,000, and so on.

In a number such as 23, the digits 2 and 3 have place value as follows:

$$23 = (2 \cdot 10) + (3 \cdot 1)$$

Now consider the same two digits with a dot, called the **decimal point,** in front of them, .23. We call this number a **decimal fraction,** or just plain **decimal.** (It is standard procedure to place a zero to the left of the decimal point if the decimal is less than 1. The zero helps the reader see the decimal point and emphasizes the fact that we are dealing with a decimal fraction.) In this new form, we have

$$0.23 = \left(2 \cdot \frac{1}{10}\right) + \left(3 \cdot \frac{1}{100}\right) = \frac{2}{10} + \frac{3}{100} = \frac{20}{100} + \frac{3}{100}$$

which can be written simply as

$$\frac{23}{100}$$

and read "twenty-three hundredths." The decimal point is placed so that the number of digits to the right of it indicates the number of zeros in the fraction's denominator. If there is one digit to the right of the decimal point, the

denominator is 10, read "tenths." If there are two digits to the right of the decimal point, the denominator is 100, read "hundredths." If three digits are to the right of the decimal point, the denominator is 1,000, read "thousandths." Four digits to the right of the decimal point are read "ten-thousandths," five digits are read "hundred-thousandths," and so on.

To read a decimal fraction: Read the whole number (if any); next read "and" for the decimal point. Then read the portion after the decimal point as a whole number. Finally, read the name of the decimal place of the last digit on the right.

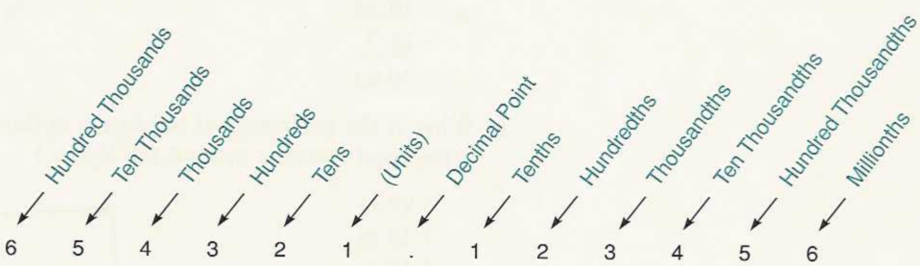
For example, 0.27, which is read "twenty-seven *hundredths,*" is written

$$0.27 = \frac{27}{100}$$ Hundred

while 0.149, which is read "one hundred forty-nine *thousandths,*" is written

$$0.149 = \frac{149}{1{,}000}$$ Thousand

The last digit in the number is the key to the denominator of the fraction.



■ Example 1–2 A

Write the following decimal numbers as fractions reduced to lowest terms.

1. 0.8 (read "eight *tenths*")

$$0.8 = \frac{8}{10}$$ Ten

$$= \frac{4}{5}$$ Reduce to lowest terms

2. 0.57 (read "fifty-seven *hundredths*")

$$0.57 = \frac{57}{100}$$ Hundred

3. 0.1234 (read "one thousand two hundred thirty-four *ten thousandths*")

$$0.1234 = \frac{1{,}234}{10{,}000}$$ Ten thousand

$$= \frac{617}{5{,}000}$$ Reduce to lowest terms

Note A decimal number that is written as a fraction will reduce *only if* the numerator is divisible by 2 or 5. This was the case in examples 1 and 3.

▶ ***Quick check*** Write 0.42 as a fraction reduced to lowest terms. ■

Addition and subtraction of decimal numbers

To add or subtract decimal numbers, we place the numbers under one another so that the decimal points line up vertically and then proceed as in adding or subtracting whole numbers. The decimal point will appear in the answer directly below where it is lined up in the problem.

■ Example 1–2 B

Add or subtract the following numbers as indicated.

1. 5.67 + 32.046 + 251.7367 + 0.92

Decimal points aligned

$$\left.\begin{array}{r} 5.67 \\ 32.046 \\ 251.7367 \\ +\ \underline{0.92} \end{array}\right\} \text{Arrange numbers in columns}$$
$$290.3727$$

2. Subtract (a) 18.7 from 39.62, (b) 4.38 from 19.2

a.
$$\begin{array}{r} \overset{8}{3\cancel{9}}.62 \\ -\underline{18.7} \\ 20.92 \end{array}$$

b.
$$\begin{array}{r} 1\overset{8}{\cancel{9}}.\overset{1}{\cancel{2}} \\ -\ \underline{4.38} \\ 14.82 \end{array}$$

3. What is the perimeter of the figure in the diagram? (Recall that the perimeter is the total distance around the figure.)

$$\begin{array}{r} 3.97\text{ m} \\ 7.39\text{ m} \\ 3.18\text{ m} \\ +\ \underline{7.83}\text{ m} \\ 22.37\text{ m} \end{array}$$

▶ ***Quick check*** Subtract 14.9 from 83.42 ■

To multiply decimal numbers

1. Multiply the numbers as if they are whole numbers (ignore the decimal points).
2. Count the number of decimal places in both factors. That is, count the number of digits to the right of the decimal point in each factor. This total is the number of decimal places the product must have.
3. Beginning at the right in the product, count off to the left the number of decimal places from step 2. Insert the decimal point. If necessary, zeros are inserted so there are enough decimal places.

■ Example 1–2 C

Multiply the following.

1. 2.36 × 0.403

$$\begin{array}{r} 2.36 \\ \underline{0.403} \\ 708 \\ \underline{944} \\ 0.95108 \end{array}
\begin{array}{l} \leftarrow \text{2 decimal places} \\ \leftarrow \text{3 decimal places} \\ \\ \\ \leftarrow \text{5 decimal places} \end{array}
\quad (2 + 3 = 5)$$

2. (18.14)(106.4)

$$\begin{array}{r l} 18.14 & \leftarrow \text{2 decimal places} \\ \underline{106.4} & \leftarrow \text{1 decimal place} \quad (2 + 1 = 3) \\ 7256 & \\ 10884 & \\ \underline{1814} & \\ 1{,}930.096 & \leftarrow \text{3 decimal places} \end{array}$$

▶ ***Quick check*** (206.1)(9.36)

To divide decimal numbers, we must identify the divisor, the dividend, and the quotient in an indicated division.

$$\begin{array}{r l} 10 & \leftarrow \text{Quotient} \\ 25\overline{)250} & \leftarrow \text{Dividend} \\ \uparrow & \text{Divisor} \end{array}$$

We now outline the procedure for dividing decimal numbers.

To divide decimal numbers

1. Change the *divisor* to a whole number by moving the decimal point to the *right* as many places as is necessary.
2. Move the decimal point in the *dividend* to the right this same number of places. If necessary, zeros are inserted so there are enough decimal places.
3. Insert the decimal point in the *quotient* directly above the new position of the decimal point in the dividend.
4. Divide as with whole numbers.

■ *Example 1–2 D*

Divide the following.

1. $360.5 \div 1.03$

a. Write the problem $1.03\overline{)360.5}$

b. Move the decimal point *two* places to the right in 1.03 and 360.5

$103\overline{)36{,}050.}$ ← Zero inserted as placeholder

c. Now divide as with whole numbers.

$$\begin{array}{r} 350. \\ 103\overline{)36{,}050.} \\ \underline{30\,9} \\ 5\,15 \\ \underline{5\,15} \\ 0 \end{array}$$

The quotient is 350.

2. If an automobile travels 429.76 miles and uses 15.8 gallons of gas, how many miles per gallon did the automobile achieve?

To determine the fuel economy, we divide the total number of miles traveled by the amount of gasoline used.

$429.76 \div 15.8$

$$
\begin{array}{r}
27.2 \\
158\overline{)4{,}297.6} \\
\underline{316} \\
1137 \\
\underline{1106} \\
316 \\
\underline{316} \\
0
\end{array}
$$

The automobile achieved 27.2 miles per gallon.

▶ ***Quick check*** $4{,}950.3 \div 5.69$

We can change a fraction into its decimal number equivalent using the following procedure.

To change a fraction to a decimal number

Divide the denominator into the numerator.

Example 1–2 E

Convert each fraction to a decimal number.

1. $\frac{3}{4}$

We divide 3 by 4. To do this, we must add zero placeholders.

$$
\begin{array}{r}
0.75 \\
4\overline{)3.00} \\
\underline{2\,8} \\
20 \\
\underline{20} \\
0
\end{array}
\longleftarrow \text{Annex zeros}
$$

Thus $\frac{3}{4} = 0.75$

2. $\frac{1}{3}$

We divide $1 \div 3$.

Continues indefinitely

$$
\begin{array}{r}
0.33\overline{3} \\
3\overline{)1.000} \\
\underline{9} \\
10 \\
\underline{9} \\
10 \\
\underline{9} \\
10
\end{array}
\longleftarrow \text{Add zeros}
$$

We can see that no matter how many zero placeholders we add, the quotient will continue to add digits of 3. This is called a **repeating decimal** (denoted by the bar placed over the last digit, or digits, that are repeating). Therefore, $\frac{1}{3} = 0.\overline{3}$. We can round a repeating decimal to as many places as are needed. We can say that $\frac{1}{3}$ is *approximately equal* to 0.333, denoted by

$$\frac{1}{3} \approx 0.333$$

▶ ***Quick check*** Convert $\frac{3}{8}$ to a decimal number. ■

Percent

We use decimal numbers extensively in our work with **percent.** The word percent means "per one hundred."

> Percent is defined to be parts per one hundred.

We use the symbol "%" to represent percent. Thus

3% means "three parts per one hundred"

or

3% means "three one hundredths."

From the above discussion,

$$3\% = \frac{3}{100} = 0.03$$

That is, we can write a percent as

1. a decimal number and
2. a fraction with denominator 100

■ ***Example 1–2 F***

Write each percent as a fraction and as a decimal number.

1. 39%
39% means "thirty-nine one hundredths."

$$39\% = \frac{39}{100} = 0.39$$

2. 123%
123% means "one hundred twenty-three one hundredths."

$$123\% = \frac{123}{100} = 1.23$$

▶ ***Quick check*** Write 241% as a fraction and as a decimal. ■

From these examples, we can see how to write a percent as a decimal number.

To write a percent as a decimal number

Move the decimal point two places to the *left* and drop the % symbol.

To write a fraction as a percent

Drop the % symbol and write the number over a denominator of 100.

To write a fraction or decimal number as a percent, we reverse the procedure.

To write a decimal number as a percent

Move the decimal point two places to the *right* and affix the % symbol.

To write a fraction as a percent

Find the decimal number equivalent of the fraction and change this decimal number to a percent.

Example 1–2 G

Write the following as decimal numbers, fractions, and percents.

1. 0.9

$0.9 = 90\%$ — Move the decimal point two places to the right and affix % symbol (Add a zero placeholder)

Since $0.9 = \frac{9}{10}$ — Write as a fraction

then $0.9 = \frac{9}{10} = 90\%$

2. 1.25

$1.25 = 125\%$ — Move the decimal point two places to the right and affix % symbol

Since $1.25 = \frac{125}{100}$ — Write as a fraction

$= \frac{5}{4}$ — Reduce to lowest terms

then $1.25 = \frac{5}{4} = 125\%$

3. $\frac{7}{8}$

Divide $7 \div 8$ to obtain the decimal equivalent. Doing this we find that

$\frac{7}{8} = 0.875$

Then $\frac{7}{8} = 0.875 = 87.5\%$ — Move the decimal point two places to the right and affix % symbol

► ***Quick check*** Write 1.75 as a fraction and a percent.

Percentage

When we find 60% of 500, we find the **percentage.** In the language of mathematics, "of" usually means the operation multiplication. Thus

$$60\% \text{ of } 500 \text{ means } 60\% \cdot 500$$

However, we cannot multiply 60% times 500. We must first change 60% to a decimal number (or a fraction) before we can perform the multiplication.

$$\begin{aligned} 60\% \text{ of } 500 &= 60\% \cdot 500 \\ &= 0.60 \cdot 500 && \text{Change 60\% to 0.60} \\ &= 300 && \text{Percentage} \end{aligned}$$

Therefore $\underset{\text{(percent)}}{\overset{\uparrow}{60\%}} \cdot \underset{\text{(base)}}{\overset{\uparrow}{500}} = \underset{\text{(percentage)}}{\overset{\uparrow}{300}}$

■ ***Example 1–2 H***

Find the following percentages.

1. 8% of 35

$8\% = 0.08$ — Change percent to a decimal number

$8\% \text{ of } 35 = 0.08 \cdot 35 = 2.8$ — Multiply

Thus 8% of 35 = 2.8

2. 224% of 50

$224\% = 2.24$ — Change percent to a decimal number

$224\% \text{ of } 50 = 2.24 \cdot 50 = 112$ — Multiply

Thus 224% of 50 = 112

3. $3\frac{1}{2}\%$ of 270

$3\frac{1}{2}\% = 3.5\%$ — $\frac{1}{2} = 0.5$ as a decimal number

$= 0.035$ — Change percent to a decimal number

$3\frac{1}{2}\% \text{ of } 270 = 0.035 \cdot 270$ — Multiply

$= 9.45$

Thus $3\frac{1}{2}\%$ of 270 = 9.45

▶ ***Quick check*** 236% of 20 ■

Mastery points

Can you

- Write decimal numbers as fractions?
- Add and subtract decimal numbers?
- Multiply and divide decimal numbers?
- Write fractions as decimal numbers?
- Change a percent to a decimal number?
- Change a decimal number to a percent?
- Change a fraction to a percent?
- Change a percent to a fraction?
- Find the percentage?

Exercise 1–2

Write each decimal number as a fraction reduced to lowest terms. See example 1–2 A.

Example 0.42

Solution 0.42 is read "forty-two *hundredths*."

$$0.42 = \frac{42}{100} \leftarrow \text{Hundred}$$
$$= \frac{21}{50} \quad \text{Reduce to lowest terms}$$

1. 0.4 **2.** 0.8 **3.** 0.15 **4.** 0.36
5. 0.125 **6.** 0.248 **7.** 0.875 **8.** 0.625

Add or subtract the following as indicated. See example 1–2 B.

Example Subtract 14.9 from 83.42

Solution We want 83.42 − 14.9

$$\begin{array}{r} \overset{7}{\cancel{8}}\overset{2}{\cancel{3}}.42 \\ -\underline{14.9} \\ 68.52 \end{array}$$

9. 6.8 + 0.354 + 2.78 + 7.083 + 2.002
10. 4.76 + 0.573 + 3.57 + 40.09 + 13
11. 8.0007 + 360.01 + 25.72 + 6.362 + 140.2
12. 7.0001 + 8 + 7.067 + 803.1 + 5.25
13. 10.03 + 3.113 + 0.3342 + 0.0763 + 0.005
14. 27.376 − 14.007
15. 367.0076 − 210.02
16. 836 − 0.367
17. 1.07 − 0.00036
18. 4,563.2 − 274.063

Multiply the following. See example 1–2 C.

Example (206.1)(9.36)

Solution

$$\begin{array}{r l} 206.1 & \leftarrow \text{1 decimal place} \\ \underline{9.36} & \leftarrow \text{2 decimal places} \quad (1 + 2 = 3) \\ 12366 & \\ 6183 & \\ \underline{18549} & \\ 1{,}929.096 & \leftarrow \text{3 decimal places} \end{array}$$

19. (7.006)(1.36) **20.** (42.6)(73) **21.** (56.37)(0.0076) **22.** 703.6 × 1.7
23. 30.0303 × 0.030303 **24.** 2.456 × 0.00012

Divide the following. See example 1–2 D.

Example 4,950.3 ÷ 5.69

Solution 1. Write the problem $5.69\overline{)4{,}950.3}$

2. Move the decimal point *two* places in 5.69 and 4,950.3

$$569\overline{)495{,}030.} \longleftarrow \text{Add zero placeholder}$$

3. Divide as whole numbers.

$$\begin{array}{r} 870. \\ 569\overline{)495{,}030.} \\ \underline{4552} \\ 3983 \\ \underline{3983} \\ 0 \end{array} \longleftarrow \text{Add zero placeholder}$$

Thus 4,950.3 ÷ 5.69 = 870

25. 0.84 ÷ 0.7
26. 0.525 ÷ 0.5
27. 10.4 ÷ 0.26
28. 21.681 ÷ 8.03
29. 6,125.1 ÷ 60.05
30. 166.279 ÷ 64.7
31. 31.50 ÷ 0.0126
32. 2.9868 ÷ 0.057

Convert each fraction to a decimal number. See example 1–2 E.

Example $\frac{3}{8}$

Solution We divide 3 ÷ 8, adding zero placeholders where necessary.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{2\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array} \longleftarrow \text{Zero placeholders}$$

The decimal equivalent of $\frac{3}{8}$ is 0.375

33. $\frac{3}{20}$
34. $\frac{5}{8}$
35. $\frac{13}{20}$
36. $\frac{17}{50}$
37. $\frac{2}{9}$
38. $\frac{5}{9}$

39. Heating oil costs 89.9 cents per gallon. What is the total cost of 14.36 gallons, correct to the nearest cent?

40. A carpenter has three pieces of wood which are 24.5 inches, 35.25 inches, and 62.375 inches long, respectively. How many inches of wood does she have all together?

41. A wood craftsman has 74.75 inches of a particular stock. He needs 5.75 inches of the stock to carve out a cardinal bird. How many cardinals can he make?

42. A rectangular field is 21.3 yards long and 15.75 yards wide. Find the area (length × width) of the field.

43. The 500-meter speed-skating event was won in a time of 43.33 seconds in the 1972 olympics. The winning time in 1976 was 42.76 seconds. How much faster was the 1976 time?

44. A student bought a book for $21.68. If she gave the cashier $25, how much change did she receive?

45. On a 4-day trip, the Adams family used 32.5 gallons, 28.36 gallons, 41.87 gallons, and 19.55 gallons of gasoline. How many gallons of gasoline did they use all together?

46. An airline pilot flew distances of 210.6 kilometers, 504.3 kilometers, 319.6 kilometers, 780.32 kilometers, and 421.75 kilometers on five flights. How many kilometers did he fly all together?

47. A rectangular field is 43.3 yards long and 25.34 yards wide. Find the area (length $\times$ width) of the field.

48. If a cubic foot of water weighs 62.5 pounds, how many pounds of water are there in a tank containing 10.4 cubic feet?

Write each percent as a fraction and as a decimal number. See example 1–2 F.

Example 241%

Solution 241% means "two hundred forty-one one hundredths."

$$241\% = \frac{241}{100} = 2.41$$

49. 5% **50.** 1% **51.** 12% **52.** 64%

53. 135% **54.** 150% **55.** 325% **56.** 570%

Write exercises 57–62 as a fraction and a percent and exercises 63–66 as a decimal and a percent. See example 1–2 G.

Example Write 1.75 as a fraction and a percent.

Solution $1.75 = 175\%$ Move decimal point two places to the right and affix % symbol

$$1.75 = \frac{175}{100}$$ Write as a fraction

$$= \frac{7}{4}$$ Reduce fraction to lowest terms

Thus $1.75 = \frac{7}{4} = 175\%$.

57. 0.8 **58.** 0.9 **59.** 0.54 **60.** 0.80 **61.** 1.15

62. 2.40 **63.** $\frac{3}{4}$ **64.** $\frac{5}{2}$ **65.** $\frac{3}{8}$ **66.** $\frac{5}{8}$

Find the following percentages. See example 1–2 H.

Example 236% of 20

Solution $236\% = 2.36$ Change percent to a decimal number

$236\% \text{ of } 20 = 2.36 \cdot 20$ Multiply

$= 47.2$

Thus $236\% \text{ of } 20 = 47.2$

67. 5% of 40 **68.** 8% of 45 **69.** 26% of 130

70. 78% of 900 **71.** 110% of 500 **72.** 240% of 60

73. City Bank pays 5.7% interest per year on its savings accounts. What is the annual interest on a savings account that has $4,500? (*Hint:* Annual Interest = Percent · Amount in savings.)

74. The sales tax on retail sales in Michigan is 4%. How much sales tax does John pay on a purchase of $250?

75. If Jane pays 5% of her weekly salary in state income tax, how much state tax does she pay if her weekly salary is $460?

76. A local retailer predicts his profit in a given year will be 116% of the previous year. What is his predicted profit for this year if last year's profit was $42,500?

77. The local shoe store is giving a 25% discount on clearance items. How much discount is there on a pair of shoes costing $34? What is the price of the shoes *after* the discount excluding sales tax?

78. A company charges $4\frac{1}{2}\%$ shipping and handling charges on all items shipped. What are the shipping and handling charges on goods that cost $70? What is the total cost for the goods?

79. A bottle of solution is 4% salt. How much salt is there in a 24-fluidounce bottle of solution?

80. Self-employed persons must pay a Social Security tax of 12%. What is the Social Security tax on earnings of $25,000? If the person is in the 28% federal income tax bracket, how much federal income tax does the person pay?

1–3 ■ The set of real numbers and the real number line

Set symbolism

Algebra is often referred to as "a generalized arithmetic." The operations of arithmetic and algebra differ only in respect to the symbols we use in working with each of them. Therefore, we will begin our study of algebra by dealing first with numbers and their properties. From this work, we will see algebra develop naturally as a generalized arithmetic.

To begin our study, we will start with a very simple, but important, mathematical concept—the idea of the **set.*** **A set is any collection of things.** This may be a collection of books, people, coins, golf clubs, and so on. In mathematics, we use the idea of a set primarily to denote a group of numbers. Any one of the things that belong to the set is called a **member** or an **element** of the set. One way we write a set is by listing the elements, separating them by commas, and including this listing within a pair of braces, { }.

■ Example 1–3 A

1. Using set notation, write the set of months that have exactly 30 days.
{April,June,September,November}
2. Using set notation, write the set of seasons of the year.
{spring,summer,fall,winter}

▶ ***Quick check*** Write the set of letters in the word "mathematics." Write the set of odd numbers between 5 and 10. ■

Note When we form a set, the elements within the set are never repeated and they can appear in any order.

*Georg Cantor (1845–1918) is credited with the development of the ideas of set theory. He described a set as a grouping together of single objects into a whole.

We use capital letters A, B, C, D, and so on, to represent a set. The symbol used to show that an element belongs to a set is the symbol $\in$, which we read "is an element of" or "is a member of." Consider the set $A = \{1,2,3,4\}$, which is read "the set A whose elements are 1, 2, 3, and 4." If we want to say that 2 is an element of the set A, this can be written symbolically as $2 \in A$.

A slash mark is often used in mathematics to negate a given symbol. Therefore if $\in$ means "is an element of the set," then $\notin$ would mean "is *not* an element of the set." To express the fact that 7 is not an element of set A, we could write $7 \notin A$.

Subset

Suppose that P is the set of people in a class and M is the set of men in the same class. It is obvious that the members of M are also members of P, so we say that M is a **subset** of P.

Definition

The set A is a subset of the set B if every element in A is also an element of B.

The symbol for subset is $\subseteq$, which we read "is a subset of." Therefore $A \subseteq B$ is read "the set A is a subset of the set B." Consider the following sets: $A = \{1,2,4\}$, $B = \{1,2,3,4,5\}$, and $C = \{1,3,5,7\}$. We observe that $A \subseteq B$ since every element in A is also an element of B. $C \not\subseteq B$ is read "the set C is *not* a subset of the set B," because not every element of C is an element of B. The set C contains the element 7, which is not an element of the set B.

Natural numbers and whole numbers

The most basic use of our number system is that of counting. We use 1, 2, 3, 4, 5, and so on, as symbols to represent the **natural** or **counting numbers.** The set of natural numbers will be denoted by N, as follows:

$$N = \{1,2,3,4,5, \cdots\}$$

The three dots tell us to continue this counting pattern indefinitely.

If we include 0 with the set of natural numbers, we have the set of **whole numbers,** W.

$$W = \{0,1,2,3,4,5, \cdots\}$$

We can use the set of whole numbers to represent physical quantities such as profit (100 dollars), room temperature (72 degrees), and distance (1,250 feet above sea level). However, with the set of whole numbers, we are not able to represent such things as losses of money, temperatures below zero, and distances below sea level. Therefore to represent such situations, we define a new set of numbers called the set of **integers.** We will denote this set by J.

Integers

We shall start by giving the natural numbers another name, the **positive integers.** We then form the *opposites,* or *negatives,* of the positive integers as follows: $-1, -2, -3, \ldots$. Combining the positive integers, the negative integers, and 0, we have the set of integers, J.

$$J = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$$

■ Example 1–3 B

Use integers to represent each of the following.

1. Bromine melts at seven degrees below zero Celsius.
   Answer: -7 degrees Celsius

Less than zero is a negative value

2. Bromine boils at fifty-nine degrees Celsius.
   Answer: 59 degrees Celsius

Greater than zero is a positive value

▶ ***Quick check*** Use integers to represent a debt of nine dollars. ■

Rational numbers

The set of integers is sufficient to represent many physical situations, but it is unable to provide an answer for the following problem. If we want to determine the miles per gallon (mpg) that our car is getting, and we find that 8 gallons of gas enable us to travel 325 miles, then our miles per gallon can be computed by dividing the number of miles by the number of gallons used.

$$\frac{325 \text{ miles}}{8 \text{ gallons}} = \frac{325}{8} \text{ mpg} = 40\frac{5}{8} \text{ mpg}$$

This value is not in the set of integers. Therefore to represent such situations, we define a new set of numbers called the set of **rational numbers,** which is denoted by Q. Recall that a quotient is an answer to a division problem. Hence $\frac{325}{8}$ is called a quotient of two integers since 325 and 8 are both integers.

Definition

A rational number is any number that can be expressed as a quotient of two integers in which the divisor is not zero.*

Other examples of rational numbers would be

$$\frac{2}{3}, -\frac{1}{2}, \frac{6}{1}, \frac{19}{5}, -\frac{23}{7}, \frac{15}{3}, \frac{0}{8}, \frac{-5}{1}$$

The decimal representation of a rational number is either a terminating or a repeating decimal. Some examples of terminating or repeating decimals are

$$\frac{1}{2} = 0.5, \quad \frac{1}{3} = 0.\overline{3}, \quad -\frac{1}{6} = -0.1\overline{6}, \quad -\frac{5}{4} = -1.25, \quad \frac{4}{33} = 0.\overline{12}$$

where a bar placed over a number or groups of numbers indicates that the number(s) repeat indefinitely.

Irrational numbers

At this point, we might feel that we now have numbers that will answer all possible physical situations. However that is not the case. Consider the following question.

What is the exact length of a side of a square whose area is 10 square units (figure 1–1)? To be able to answer this question, we need to find that number such that when it is multiplied with itself, the product is 10. If we use 3.16, the result would be $(3.16) \times (3.16) = 9.9856$. This is close to 10 but is not equal to 10.

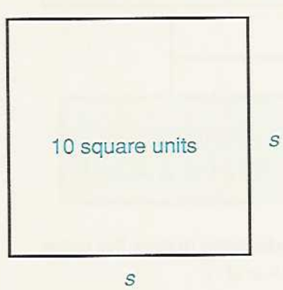


Figure 1–1

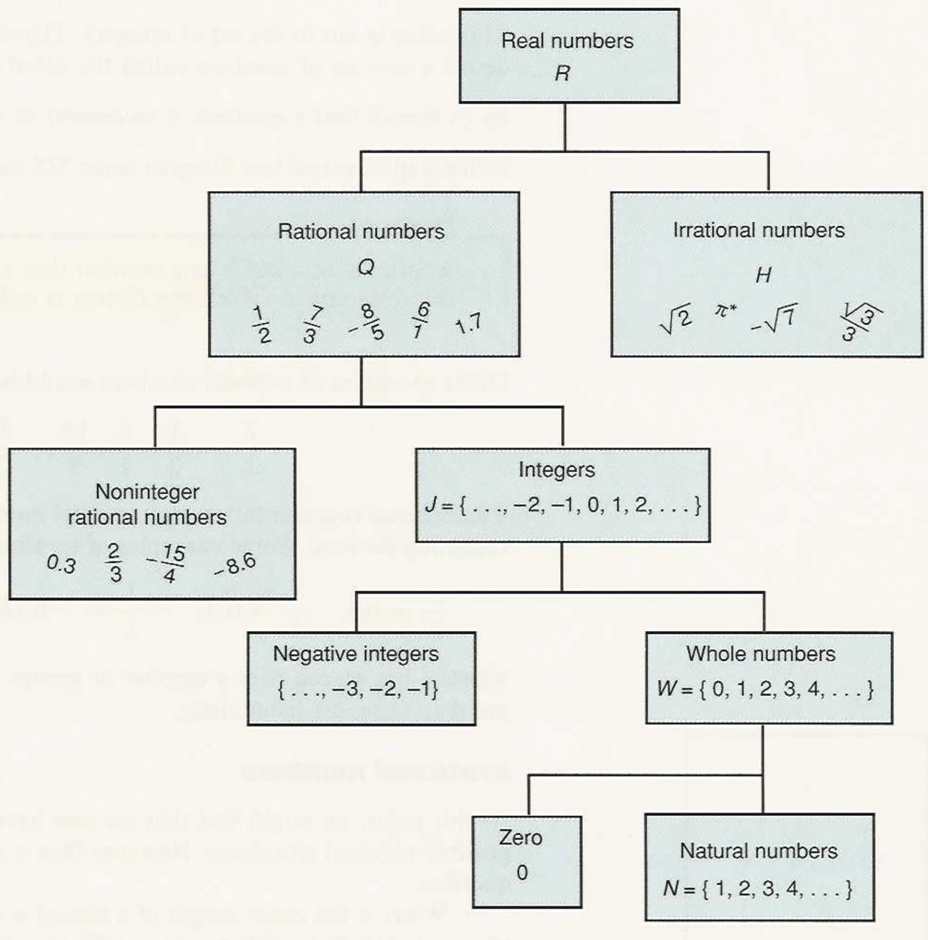
*Division involving zero will be discussed in section 1–7.

It can be shown that there is no rational value that when multiplied with itself has a product of 10. The answer to this question and many others cannot be found in the set of rational numbers. In chapter 9, we will see that the answer to this question is $\sqrt{10}$ (read "the square root of 10"). Such numbers that cannot be expressed as the quotient of two integers belong to the set of **irrational numbers,** which is denoted by H

Real numbers

Since a rational number can be expressed as the quotient of two integers and an irrational number cannot, we should realize that a number can be rational or irrational, but it cannot be both. The set that contains all of the rational numbers and all of the irrational numbers is called the set of **real numbers,** which is denoted by R. Whenever we encounter a problem and a specific set of numbers is not indicated, it will be understood that we are dealing with real numbers.

All of the sets that we have examined thus far are subsets of the set of real numbers. Figure 1–2 shows the relationship.



*π (pi) is the distance around a circle (circumference) divided by the distance across the circle through its center (diameter). Common approximations for π are 3.14 and $\frac{22}{7}$.

Figure 1–2

The real number line

To picture the set of real numbers, we shall use a real number line. We begin by drawing a line where the arrowhead at each end of the line indicates that the line continues on indefinitely in both directions. Next we choose any point on the line to represent 0. This point is called the **origin** of the number line. Numbers to the right of zero are positive and to the left of zero are negative (figure 1–3).

Figure 1–3

Any real number can now be located on the number line. Consider the number line in figure 1–4.

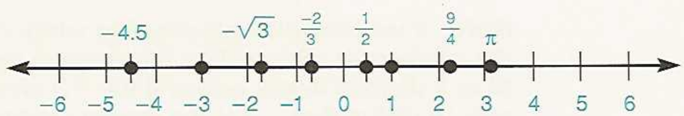


Figure 1–4

The number that is associated with each point on the line is called the **coordinate** of the point. The solid circle that is associated with each number is called the **graph** of that number. In figure 1–4, the numbers -4.5, -3, $-\sqrt{3}$, $\dfrac{-2}{3}$, $\dfrac{1}{2}$, 1, $\dfrac{9}{4}$, and π are the *coordinates* of the points indicated on the line by solid circles. The solid circles are the *graphs* of these numbers.

Note The coordinates $-\sqrt{3}$ and π represent irrational numbers. To graph these points, we would find a rational approximation using a calculator. $-\sqrt{3} \approx -1.732$, $\pi \approx 3.142$ ($\approx$ is read "is approximately equal to").

The direction in which we move on the number line is also important. If we move to the right, we are moving in a positive direction and the numbers are *increasing.* If we move to the left, we are moving in a negative direction and the numbers are *decreasing* (figure 1–5).

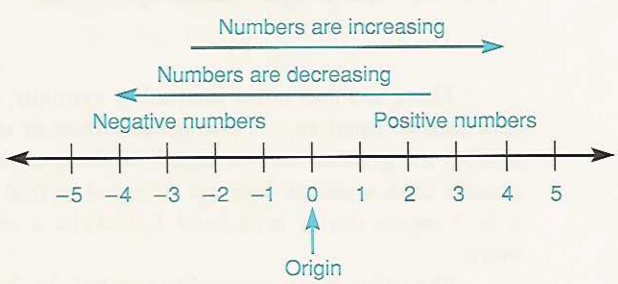


Figure 1–5

Order on the number line

So that the discussion can be more general, we will now introduce the concept of a **variable. A variable is a symbol (generally a lowercase letter) that represents an unspecified number.** A variable holds a position for a number.

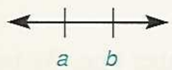


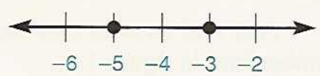
Figure 1–6

If we choose any two points on the number line and represent them by a and b, where a and b represent some *unspecified* numbers, we observe that there is an *order* relationship between a and b (figure 1–6). Since the point associated with a is to the *left* of the point associated with b, we say that a is **less than** b, which in symbols is $a < b$. We might also say that b is **greater than** a, which in symbols is $b > a$. The symbols $<$ (less than) and $>$ (greater than) are inequality symbols called **strict inequalities** and they denote an **order relationship** between numbers.

■ Example 1–3 C

Replace the ? with the proper inequality symbol ($<$ or $>$).

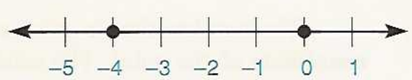
1. $-5 \ ? \ -3$



Answer: $-5 < -3$ Because -5 is to the left of -3

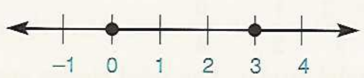
Note If we have difficulty deciding which of two numbers is greater, think of the numbers as representing temperature readings. The -5 would be thought of as 5 degrees below zero and the -3 would be 3 degrees below zero. It is easy to realize that -3 is the greater (warmer) temperature and the inequality would be $-5 < -3$.

2. $0 \ ? \ -4$



Answer: $0 > -4$ Because 0 is to the right of 4

3. $0 \ ? \ 3$

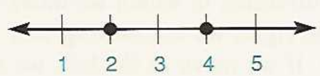


Answer: $0 < 3$ Because 0 is to the left of 3

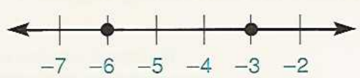
Note No matter which inequality symbol we use, the arrow *always* points at the lesser number.

► ***Quick check*** Replace ? with $<$ or $>$.

$2 \ ? \ 4$



$-3 \ ? \ -6$



■

There are two other inequality symbols, called **weak inequalities.** They are **less than or equal to,** $\leq$, and **greater than or equal to,** $\geq$. The weak inequality symbol $\geq$, greater than or equal to, denotes that one number could either be greater than a second number or equal to that second number. The inequality $x \geq 3$ means that x is *at least* 3. That is, x represents all numbers that are 3 or more.

The other weak inequality symbol, $\leq$, less than or equal to, denotes that one number could either be less than a second number or equal to that second number. The inequality $x \leq 5$ means that x is *at most* 5. That is, x represents all numbers that are 5 or less.

Absolute value

As we study the number line, we observe a very useful property called **symmetry.** The numbers are symmetrical with respect to the origin. That is, if we go four units to the right of 0, we come to the number 4. If we go four units to the left of 0, we come to the *opposite* of 4, which is -4 (figure 1–7).

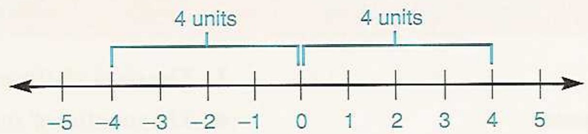


Figure 1–7

Each of these numbers is four units away from the origin. How far a given number is from the origin is called the **absolute value** of the number. **The absolute value of a number is the undirected distance that the number is from the origin.** The symbol for absolute value is $|\ \ |$.

■ Example 1–3 D

Evaluate the following expressions.

1. $|-7| = 7$ **2.** $|3| = 3$ **3.** $|0| = 0$

4. $\left|\dfrac{2}{3}\right| = \dfrac{2}{3}$ **5.** $|-3.7| = 3.7$ **6.** $-|-6| = -6$

Note The absolute value of a number is *never* negative; that is, $|x| \geq 0$, for every $x \in R$, however the absolute value bars are only applied to the symbol contained within them. The $-$ sign in front of the absolute value bars is not affected by the absolute value bars. Example 6 would be read "the opposite of the absolute value of -6," and the answer is -6.

▶ ***Quick check*** Evaluate the following expressions.
$|-3|$; $|12|$; $-|-4|$ ■

Visualizing our number system on the number line demonstrates the fact that each number possesses two important properties.

1. The sign of the number denotes a direction from zero. (The absence of a sign indicates a positive number.)
2. The absolute value represents a distance from zero.

Mastery points

Can you

- Write sets?
- Draw a number line?
- Graph a number on the number line?
- Tell which of two real numbers is greater?
- Find absolute values?
- Approximate the value of the coordinate of a graph on the number line?

Exercise 1–3

Write each set by listing the elements. See example 1–3 A.

Examples	The letters in the word "mathematics"	The odd numbers between 5 and 10
Solutions	{m,a,t,h,e,i,c,s} — The letters *m*, *a*, and *t* are not repeated within the set	{7,9} — The word between means that we do not include the 5 or 10

1. The days of the week
2. The days of the week that begin with "T"
3. The first 3 months of the year
4. The months of the year that begin with "J"
5. The months with 31 days
6. The letters in the word "repeat"
7. The letters in the word "algebra"
8. The letters in the word "elementary"
9. The letters in the word "intermediate"
10. The even numbers between 5 and 11
11. The odd numbers between 2 and 10
12. The months of the year that begin with "A"
13. The days of the week that begin with "S"
14. The months of the year that begin with "M"

Use integers to represent each of the following. See example 1–3 B.

Example A debt of nine dollars

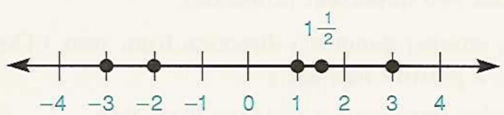
Solution -9 dollars — A debt is a negative value

15. Ten dollars overdrawn in a checking account; 150 dollars in a savings account
16. Mercury's melting point is 39 degrees below zero Celsius. Its boiling point is 357 degrees Celsius.
17. A loss of 10 yards on a football play; a gain of 16 yards
18. Mt. Everest rises 29,028 feet above sea level; the Dead Sea has a depth of 1,290 feet below sea level.
19. The Dow Jones Industrial Stock Average fell 14 points; it rose 8 points.
20. Hydrogen's melting point is 259 degrees below zero Celsius. Water boils at 100 degrees Celsius. Water freezes at zero degrees Celsius.

Plot the graph of the following numbers, using a different number line for each exercise. See figure 1–4.

Example $-3, -2, 1, 1\frac{1}{2}, 3$

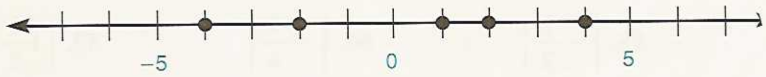
Solution



21. $-3, -1, \frac{1}{2}, 3, 4$
22. $-2, -1, \frac{3}{4}, 2, 4$
23. $-5, -2, 0, 1, 3\frac{1}{2}$
24. $-1, 0, \frac{2}{3}, 1, 3$
25. $-4, -2, 1, 2\frac{1}{2}, 4$
26. $-3, -1, \frac{1}{2}, 1, 2$
27. $\boxed{27.}$ $-5, -1, 0, \frac{1}{2}, \sqrt{2}, 6$
28. $-4, -\sqrt{4}, \frac{1}{2}, \sqrt{3}, \pi$

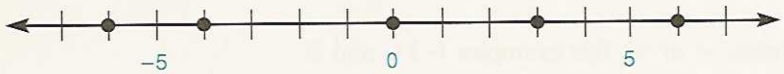
Approximate the values of the set of coordinates of the graphs on the following number lines to the nearest $\frac{1}{4}$ of a unit. See figure 1–4.

Example



Solution $-4, -2, 1, 2, 4$

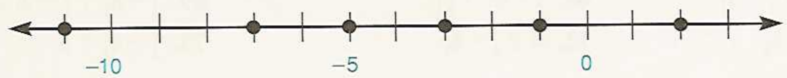
29.



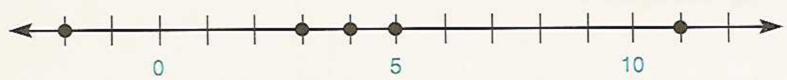
30.

31.

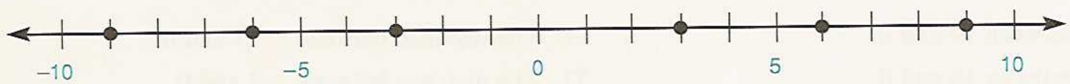
32.



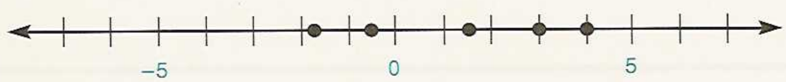
33.



34.



35.



36.

Replace the ? with the proper inequality symbol, $<$ or $>$. See example 1–3 C.

Examples $2 \ ? \ 4$ $\qquad -3 \ ? \ -6$

Solutions $2 < 4$ Because 2 is to the left of 4 on the number line $\qquad -3 > -6$ Because -3 is to the right of -6 on the number line

37. $4 \ ? \ 8$ **38.** $6 \ ? \ 3$ **39.** $9 \ ? \ 2$ **40.** $-2 \ ? \ -4$ **41.** $-3 \ ? \ -8$
42. $-9 \ ? \ -6$ **43.** $-10 \ ? \ -5$ **44.** $0 \ ? \ 2$ **45.** $0 \ ? \ 4$ **46.** $-3 \ ? \ 0$
47. $0 \ ? \ -6$

Evaluate the following expressions. See example 1–3 D.

Examples $|-3|$ $\qquad |12|$ $\qquad -|-4|$

Solutions 3 -3 is 3 units from the origin $\qquad 12$ 12 is 12 units from the origin $\qquad -4$ -4 is 4 units from the origin. The problem is to find the opposite of the absolute value

48. $|0|$ **49.** $|2|$ **50.** $|8|$ **51.** $|-5|$ **52.** $|-7|$

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** Undergraduate students may choose to defer repayment until six months after graduation or ceasing to be enrolled at least half time in school. Interest only and immediate repayment options also available.

*** A 0.25% interest rate reduction is available for borrowers who elect to have monthly principal and interest payments transferred electronically from a savings or checking account. The interest rate reduction will begin when automatic principal and interest payments start, and will remain in effect as long as automatic payments continue without interruption. This reduced interest rate will return to contract rate if automatic payments are cancelled, rejected or returned for any reason. Upon request, borrowers are also entitled to an additional 0.25% interest rate reduction if (1) the first 36 payments of principal and interest are paid on time, and (2) at any time prior to the 36th on time payment, the borrower who receives the monthly bill elects to have monthly principal and interest payments transferred electronically from a savings or checking account, and continues to make such automatic payments through the 36th payment. This reduced interest rate will not be returned to contract rate if, after receiving the benefit, the borrower discontinues automatic electronic payment. The lender and servicer reserve the right to modify or discontinue borrower benefit programs (other than the co-signer release benefit) at any time without notice.

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53. $|4|$ **54.** $\left|\frac{2}{3}\right|$ **55.** $\left|-\frac{1}{2}\right|$ **56.** $\left|-\frac{3}{4}\right|$ **57.** $\left|1\frac{1}{2}\right|$

58. $-\left|-2\frac{3}{4}\right|$ **59.** $-\left|\frac{5}{8}\right|$ **60.** $-|-2|$ **61.** $-|6|$

Replace the ? with the proper inequality symbol, $<$ or $>$. See examples 1–3 C and D.

Examples $|-6| ? |-3|$ $|4| ? |-7|$

Solutions $6 ? 3$ $|-6|$ is 6 and $|-3|$ is 3

$6 > 3$ 6 is to the right of 3 on the number line

then $|-6| > |-3|$

$4 ? 7$ $|4|$ is 4 and $|-7|$ is 7

$4 < 7$ 4 is to the left of 7 on the number line

then $|4| < |-7|$

62. $|-2| ? |-4|$ **63.** $|5| ? |-7|$ **64.** $|-3| ? |-4|$ **65.** $|0| ? |-2|$

66. $|-3| ? |4|$ **67.** $|-8| ? |-5|$ **68.** $|-9| ? |7|$ **69.** $|-6| ? |-2|$

70. $|-5| ? 3$ **71.** $7 ? |-2|$ **72.** $4 ? |-8|$ **73.** $|-4| ? 6$

Use absolute value to write each of the following. See figure 1–7.

Example The distance between -8 and 0

Solution $|-8|$ The absolute value of a number is the distance from the number to the origin.

74. The distance between 14 and 0

75. The distance between -27 and 0

76. The distance between 18 and 0

77. The distance between -9 and 0

78. The distance between -19 and 0

1–4 ■ *Addition of real numbers*

Addition of two positive numbers

When we perform the operations of addition and subtraction with integers, we will refer to this as operations with **signed numbers.** We use the minus sign $(-)$ to indicate a negative number and the plus sign $(+)$ to indicate a positive number. We should realize that the minus sign is identical to the symbol used for subtraction, and the plus sign is identical to the symbol used for addition. The meanings of these symbols will depend on their use in the context of the problem. In the case of a positive number, the plus sign need only be used if we wish to emphasize the fact that the number is positive. *When there is no sign, the number is understood to be positive.*

To visualize the idea of addition of signed numbers, we will use the number line to represent a checking account in which the origin represents a zero balance. We will let moves in the positive direction represent deposits and moves in the negative direction represent checks that we write, withdrawals. If we have a zero balance and deposit 5 dollars and 4 dollars, represented by $(+5)$ and $(+4)$, the balance in the account would be as shown in figure 1–8.

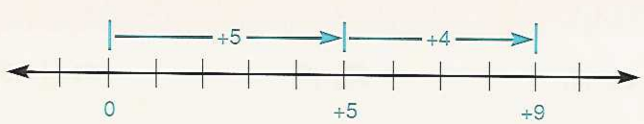

Figure 1–8

Performing the addition, we have $(+5) + (+4) = (+9)$. Notice that the sum, $(+9)$, has the same sign as the 5 and the 4.

So the discussion can be more general, we will use variables to state the process of addition on the number line. Recall that a variable represents an unspecified number. It is a placeholder for a number.

Addition on the number line

To add a and b, that is $a + b$, we locate a on the number line and move from there according to b.

1. If b is positive, we move to the right b units.
2. If b is negative, we move to the left the absolute value of b units.
3. If b is 0, we stay at a.

■ Example 1–4 A

Add the following numbers.

1. $(+4) + (+5) = +9$ 4 and 5 are called addends, 9 is the sum

2. $(+3) + (+8) = +11$ The sum of 3 and 8 is 11

3. $(+6) + (0) = +6$ 6 plus 0 equals 6 ■

We have used the plus $(+)$ sign in front of a number to emphasize the fact that the number was positive. In future examples, we will omit the plus sign from a positive number and it will be understood that 3 means $+3$.

From example 3, we see that when zero and a given number are added, the sum is the given number. For this reason, zero is called the **identity element of addition.** We now state this property.

Identity property of addition

For every real number a,

$$a + 0 = 0 + a = a$$

Concept

Adding zero to a number leaves the number unchanged.

Observe that our example in figure 1–8 and example 1 in example 1–4 A are the same addition problem with the order of the numbers reversed. That is, $(+5) + (+4) = +9$ and $(+4) + (+5) = +9$. This observation illustrates an important mathematical principle called the **commutative property of addition.**

Commutative property of addition

For every real number a and b,

$$a + b = b + a$$

Concept

This property says that when we are *adding* numbers, changing the order in which the numbers are added will not change the answer (sum).

Addition of two negative numbers

We now examine addition of two negative numbers. If we have a zero balance and write a check for 6 dollars, expressed as (-6), and another for 5 dollars, expressed as (-5), the loss to our checking account would be as shown in figure 1–9.

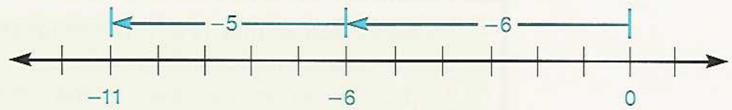

Figure 1–9

Our total withdrawal would be (-6) dollars $+$ (-5) dollars $= -11$ dollars.

We can summarize what we have done by saying: *When we add two negative numbers, we add their absolute values and prefix the sum with their common sign, −.*

■ Example 1–4 B

Add the following numbers.

1. $(-2) + (-7) = -9$
2. $(-9) + (-4) = -13$
3. $(-20) + (-30) = -50$
4. $(-6) + (-11) = -17$

▶ ***Quick check*** $(-5) + (-7)$ ■

We see from our examples that when we add two signed numbers and their signs are the same, we add their absolute values and prefix the sum with their common sign.

Addition of two numbers with different signs

To consider the addition of two numbers with different signs, we again refer to our checking account. Suppose we make a deposit to and a withdrawal from our checking account. If we deposit more money than we withdraw, we will have a positive balance in our account. For example, if we have a zero balance and deposit 15 dollars, represented by $(+15)$ dollars, and withdraw 10 dollars, represented by (-10) dollars, the result to our checking account will be as shown in figure 1–10.

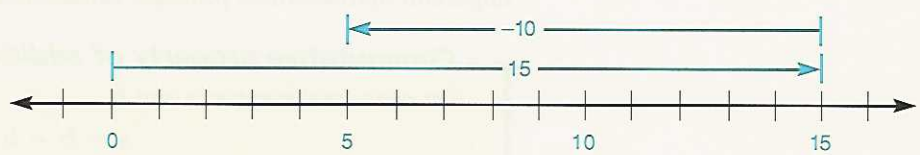

Figure 1–10

The balance in the checking account would be $(+15)$ dollars $+$ (-10) dollars $= (+5)$ dollars.

We see that our answer is the absolute value of the difference of 15 and 10, prefixed by the sign of the number with the greater absolute value, $(+15)$.

Consider a second example in which we have a zero balance and deposit 10 dollars, $(+10)$ dollars, and write a check for 15 dollars, (-15) dollars. The result to our checking account this time would be as shown in figure 1–11.

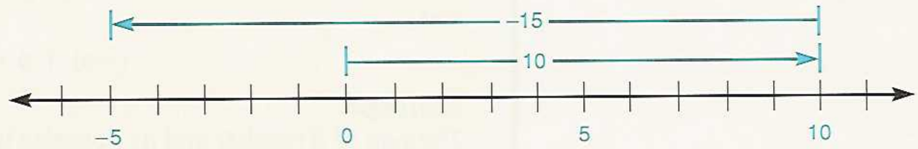

Figure 1–11

The balance would be $(+10)$ dollars $+ (-15)$ dollars $= (-5)$ dollars.

We again see that our answer is the absolute value of the difference of the two numbers prefixed by the sign of the number with the greater absolute value, (-15).

We now summarize the procedure for adding two numbers of different signs. *The sum of a positive number and a negative number is found by subtracting the lesser absolute value from the greater absolute value. The answer has the sign of the number with the greater absolute value.*

■ Example 1–4 C

Add the following numbers.

When numbers have the same sign
Sum has their common sign
Add their absolute values

1. $(-6) + (-10) = -16$

$$\left.\begin{array}{r} |-6| \\ +\underline{|-10|} \end{array}\right\} \quad \begin{array}{r} 6 \\ \underline{+10} \\ 16 \end{array}$$

When numbers have different signs
Sum has sign of number with the greater absolute value, -8
Subtract the lesser absolute value from the greater absolute value

2. $(-8) + (5) = -3$

$$\left.\begin{array}{r} |-8| \\ -\underline{|5|} \end{array}\right\} \quad \begin{array}{r} 8 \\ \underline{-5} \\ 3 \end{array}$$

3. $(-4) + (6) = 2$ — Difference of the absolute values and the sign comes from the $+6$

4. $(10) + (-10) = 0$ — The sum of a number and its opposite is zero

▶ ***Quick check*** $(3) + (-8)$ ■

We observe from example 4 that $(10) + (-10) = 0$. That is, a number added to its opposite gives a sum equal to zero. The opposite of a number is also called the **additive inverse** of that number. We now state this property.

Additive inverse property

For every real number a,

$$a + (-a) = 0$$

and

$$(-a) + a = 0$$

Concept

The sum of a number and its opposite is zero. Its opposite is called its additive inverse.

We can summarize our procedure for addition of real numbers as follows:

Addition of two real numbers

1. If the signs are the same, we add their absolute values and prefix the sum by their common sign.
2. If the signs are different, we subtract the lesser absolute value from the greater absolute value. The answer has the sign of the number with the greater absolute value.
3. The sum of a number and its opposite (additive inverse) is zero.

In many problems, there will be more than two numbers being added together. In those situations, as long as the operation involved is *strictly addition,* we can add the numbers in any order we wish.

■ *Example 1–4 D*

Find the sum.

1. $(-2) + (-7) + (12) = (-9) + (12) = 3$

First $(-2) + (-7)$ is -9
Then $(-9) + (12)$ is 3

Note For convenience, we simply add the numbers as they appear, reading them from left to right.

2. $(14) + (-4) + (4) = (10) + (4) = 14$

First $(14) + (-4)$ is 10
Then $(10) + (4)$ is 14 ■

Since the numbers in the preceding examples can be added in any order, we might feel in example 2 that it would be easier to add the (-4) and (4) first, as follows:

$$(14) + (-4) + (4) = (14) + 0 = 14$$

Therefore we observe the following:

$$[(14) + (-4)] + (4) = (14) + [(-4) + (4)] = 14$$

This illustrates a mathematical principle called the **associative property of addition.**

Associative property of addition

For every real number a, b, and c,

$$(a + b) + c = a + (b + c)$$

Concept

Changing the grouping of the numbers will not change the sum.

Problem solving

To solve the following word problems, we must find the sum of the given quantities. Represent gains by positive integers and losses by negative integers.

■ Example 1–4 E

Choose a variable to represent the unknown quantity and find its value.

1. A pipe that is 4 feet long is joined with a pipe that is 6 feet long. What is the total length of the pipe?

   Let ℓ = the total length of the pipe. To find the total length, we must *add* the individual lengths.

total length	is	4-foot pipe	joined with	6-foot pipe
ℓ	$=$	4	$+$	6

   $\ell = 4 + 6$
   $\ell = 10$

   The total length of the pipe is 10 feet.

2. The quarterback of the Detroit Lions attempted 4 passes with the following results: a 12-yard gain, an incomplete pass, a 5-yard loss (tackled behind the line), and a 15-yard gain. What was his total gain (or loss)?

   Let t = the total gain (or loss). To find the total gain (or loss), we must *add* the results of the 4 plays. Then

total gain (or loss)	is	12-yard gain		incomplete pass		5-yard loss		15-yard gain
t	$=$	(12)	$+$	(0)	$+$	(-5)	$+$	(15)

   $t = (12) + (0) + (-5) + (15)$
   $t = (12) + (-5) + (15) = (7) + (15) = 22$

   The total *gain* was 22 yards after the 4 plays. ■

Mastery points

Can you

- Add real numbers on the number line?
- Add real numbers mentally?
- Use the commutative and associative laws of addition?

Exercise 1–4

Find each sum. See examples 1–4 A, B, C, and D.

Examples	$(-5) + (-7)$		$(3) + (-8)$
Solutions	-12 Signs are the same, add their absolute values and prefix the sum by their common sign		-5 Signs are different, subtract lesser absolute value from greater, sign comes from number with the greater absolute value

1. $(-9) + (-4)$	**2.** $(+3) + (-5)$	**3.** $(+7) + (-2)$	**4.** $(-14) + (-10)$
5. $(-8) + 3$	**6.** $(+12) + (-8)$	**7.** $4 + (-9)$	**8.** $(-11) + 7$
9. $4 + (-4)$	**10.** $(-3) + 3$	$\boxed{\textbf{11.}}$ $(-8.7) + (-4.9)$	**12.** $(-12.1) + 8.6$

13. $(-3.7) + (-7.4)$
14. $(-8.3) + (15.8)$
$\boxed{15.}$ $\left(-\frac{1}{6}\right) + \left(-\frac{1}{3}\right)$
16. $\frac{3}{4} + \left(-\frac{7}{8}\right)$
17. $\frac{1}{5} + \left(-\frac{1}{10}\right)$
18. $\left(-1\frac{1}{2}\right) + \left(-1\frac{1}{4}\right)$
19. $\left(2\frac{1}{2}\right) + \left(-3\frac{1}{4}\right)$
20. $\left(5\frac{3}{8}\right) + \left(-2\frac{1}{4}\right)$
$\boxed{21.}$ $10 + (-5) + (-2)$
22. $3 + (-4) + 1$
$\boxed{23.}$ $(-12) + (-10) + (+8) + (+24)$
24. $(-24) + (+12) + (+12)$
25. $(-30) + 14 + (-8) + (-20)$
26. $(-2) + (+3) + (-4) + (-5)$
27. $(-25) + 4 + (-32) + 28 + 3$
28. $(+11) + (-12) + (-14) + (-9)$

Find the sum.

Example The sum of -6, 2, and -9

Solution $(-6) + 2 + (-9) = (-4) + (-9)$ First add -6 and 2
$= -13$ Then add -4 and -9

29. The sum of 7, -11, and -6
30. The sum of -16, -6, and -5
31. 18 plus -14 plus -4
32. -9 plus 15 plus -17
33. The sum of -5 and 12 increased by 4
$\boxed{34.}$ The sum of 15 and -18 increased by 10
35. 15 added to the sum of -9 and 9
36. 9 added to the sum of -6 and -11

See example 1–4 E.

Example If a man borrows \$1,800 and has \$700 in savings, what is his net worth?

Solution We represent the borrowed amount as $(-1{,}800)$ dollars and his savings as $(+700)$ dollars. His net worth is then represented by $(-1{,}800)$ dollars $+$ $(+700)$ dollars $=$ $(-1{,}100)$ dollars.

37. A temperature of $(-18)°$ C is increased by 25° C. What is the resulting temperature?
38. Tim Wesner received money for his birthday from 4 different people. He received \$10, \$8, \$7, and \$5. How much money did Tim receive for his birthday?
39. Jane Balch made profits of \$5, \$7, \$2, \$3, and \$15 in five days of selling Kool Aid. How much did she receive from her five-day sale?
40. The stock market rose by 23 points on Monday, fell by 10 points on Tuesday, rose by 8 points on Wednesday, rose by 31 points on Thursday, and fell by 19 points on Friday. What was the total gain (or loss) during the 5 days?
41. The barometric pressure rose 6 mb (millibars), then dropped 9 mb. Later that day the pressure dropped another 3 mb and then rose 8 mb. What was the total gain (or loss) in barometric pressure that day?
42. A small business showed profits of \$15, \$25, \$10, \$9, and \$27 on five consecutive days. What was the total profit?
43. John's blood pressure was 118. It changed by -19. Find his present blood pressure.
$\boxed{44.}$ The temperature in Sault Ste. Marie was $-13°$ F at 8 A.M. By 1 P.M. that day, it had risen 39° F. What was the temperature at 1 P.M.?
45. The temperature on a given day in Anchorage, Alaska, was $-19°$ F. The temperature then went down 22° F. What was the final temperature that day?
46. A TWA plane is flying at an altitude of 33,000 feet. It suddenly hits an air pocket and drops 4,200 feet. What is its new altitude?
47. Mack Wooten has \$35 in his checking account. He deposits \$52, \$25, and \$32; he then writes checks for \$18 and \$62. What is the final balance in his checking account?
48. A football team has the ball on its 25-yard line. On three successive plays, the team gains 6 yards, loses 3 yards, and then gains 4 yards. Where does the ball rest for the fourth play?

1–5 ■ Subtraction of real numbers

Subtraction of two real numbers

We are already familiar with the operation of subtraction in problems such as $15 - 10 = 5$. From the definition of subtraction, we know that $15 - 10 = 5$ since $5 + 10 = 15$. In figure 1–6 of section 1–4, we see that $15 + (-10)$ is also equal to 5. That is,

$$\overset{\text{Subtraction}}{15 - 10 = 5} \quad \text{and} \quad \overset{\text{Addition}}{15 + (-10) = 5}$$

From this example, we see that we obtain the same results if we change the operation from subtraction to addition and change the sign of the number that we are subtracting. We would have an addition problem and could proceed as we did in section 1–4.

To summarize our procedure for subtracting real numbers, we can state algebraically:

Definition of subtraction

For any two real numbers, a and b,

$$a - b = a + (-b)$$

Concept

"a minus b" means the same as "a plus the opposite of b."

Opposite of b

$$(a) - (b) = (a) + (-b)$$

Change to addition

Our steps to carry out the subtraction would be as follows:

Subtraction of two real numbers

Step 1 We change the operation from subtraction to addition.
Step 2 We change the sign of the number that follows the subtraction symbol.
Step 3 We perform the addition, using our rules for adding signed numbers.

■ Example 1–5 A

Subtract the following numbers.

		Step 1 Subtraction to addition	***Step 2*** Change sign of number being subtracted		***Step 3*** Add
1. $(9) - (5) =$	(9)	$+$	(-5)	$=$	4
2. $(4) - (11) =$	(4)	$+$	(-11)	$=$	-7
3. $(-9) - (5) =$	(-9)	$+$	(-5)	$=$	-14

	Step 1 Subtraction to addition		*Step 2* Change sign of number being subtracted		*Step 3* Add
4. $(6) - (-8) =$	(6)	$+$	(8)	$=$	14
5. $(-12) - (-8) =$	(-12)	$+$	(8)	$=$	-4
6. $(-5) - (-14) =$	(-5)	$+$	(14)	$=$	9
7. $(8) - (5) =$	(8)	$+$	(-5)	$=$	3
8. $(5) - (8) =$	(5)	$+$	(-8)	$=$	-3

Note From examples 7 and 8, we see that the operation of subtraction is *not* commutative. That is,

$$(8) - (5) \neq (5) - (8)$$

▶ ***Quick check*** $(4) - (-6)$; $(-2) - (-8)$

Addition and subtraction of more than two real numbers

When several numbers are being added and subtracted in a horizontal line, do the problem in order from left to right. For example, in

$$9 - 3 + 4 + 3 - 6 - 1 + 4$$

	Operation being performed
$= \underline{9 - 3} + 4 + 3 - 6 - 1 + 4$	$9 - 3 = 6$
$= \underline{6 + 4} + 3 - 6 - 1 + 4$	$6 + 4 = 10$
$= \underline{10 + 3} - 6 - 1 + 4$	$10 + 3 = 13$
$= \underline{13 - 6} - 1 + 4$	$13 - 6 = 7$
$= \underline{7 - 1} + 4$	$7 - 1 = 6$
$= \underline{6 + 4}$	$6 + 4 = 10$
$= 10$	

Note If we had changed each of the indicated subtractions to addition, $9 + (-3) + 4 + 3 + (-6) + (-1) + 4$, then the order in which the problem was carried out would not change the answer. For example, $9 - 3 \neq 3 - 9$, but $9 + (-3) = (-3) + 9$.

Grouping symbols

Many times, part of the problem will have a group of numbers enclosed with grouping symbols, such as parentheses ( ), brackets [ ], or braces { }. *If any quantity is enclosed with grouping symbols, we treat the quantity within as a single number.* Thus, in

$$9 - (3 + 2) + (6 - 2) - (5 - 4)$$

we perform operations within parentheses first to get

$$9 - 5 + 4 - 1$$

which gives

$$4 + 4 - 1 = 8 - 1 = 7$$

Then

$$9 - (3 + 2) + (6 - 2) - (5 - 4) = 7$$

■ Example 1–5 B

Perform the indicated operations.

1. $$\begin{aligned} 8 - 3 + 2 - 5 - 1 &= 5 + 2 - 5 - 1 \\ &= 7 - 5 - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

2. $$\begin{aligned} [6 + 1] - [2 - 5] + 7 + [9 - 6] &= 7 - [-3] + 7 + 3 \\ &= 10 + 7 + 3 \\ &= 17 + 3 \\ &= 20 \end{aligned}$$

3. $(14 - 7) - 2 = 7 - 2 = 5$

4. $14 - (7 - 2) = 14 - 5 = 9$

Note We observe from examples 3 and 4 that the operation of subtraction is *not* associative. That is, order does make a difference in subtraction.

$$(14 - 7) - 2 \neq 14 - (7 - 2)$$

▶ ***Quick check*** $14 - 11 + 18 - (7 - 12) + 2$ ■

Problem solving

To solve the following word problems, we must find the difference between two quantities. To find the difference we must *subtract*.

■ Example 1–5 C

Choose a letter for the unknown and find its value by subtracting.

1. On a given winter's day in Detroit, Michigan, the temperature was 31° in the afternoon. By 9 P.M. the temperature was −12°. How many degrees did the temperature drop from afternoon to 9 P.M.?

Let t = the number of degrees fall in temperature. We must find the difference between 31° and −12°. Thus

degrees fall	is	difference between 31° and −12°
t	$=$	$31 - (-12)$

$t = 31 - (-12) = 31 + 12 = 43$

There was a 43° drop in temperature.

2. From a board that is 16 feet long, John must cut a board that is 7 feet long. How much is left of the original board?

Let f = the number of feet of board left. We must find the difference between 16 and 7. Thus

feet left	is	difference between 16 and 7
f	$=$	$16 - 7$

$f = 16 - 7 = 9$

John has 9 feet of the original board left.

▶ ***Quick check*** A temperature of 14° Celsius is decreased by 18 degrees Celsius. What is the resulting temperature? ■

Mastery points

Can you

- Subtract real numbers?
- Add and subtract in order from left to right?
- Remember that subtraction is *not* commutative or associative?
- Remember that quantities within grouping symbols represent a single number?

Exercise 1–5

Find each sum or difference. See examples 1–5 A and B.

Examples		Solutions Step 1		Step 2		Step 3
$(4) - (-6)$	$=$	(4)	$+$	(6)	$=$	10
$(-2) - (-8)$	$=$	(-2)	$+$	(8)	$=$	6

1. $(4) - (5)$
2. $(-6) - (2)$
3. $(4) - (-2)$
4. $(-3) - (-7)$
5. $(-8) - (4)$
6. $(4) - (-8)$
7. $(4) - (9)$
8. $(-8) - (-5)$
9. $(-8) - (-4)$
10. $(-12) - (-16)$
11. $(8) - (-6)$
12. $(14) - (4)$
13. $(-6) + 0$
14. $(9) - (11)$
15. $(7) - 0$
16. $(6) + (-10)$
17. $\left(-\frac{1}{2}\right) - \left(-\frac{1}{4}\right)$
18. $\left(-\frac{2}{3}\right) - \left(-\frac{1}{4}\right)$
19. $1\frac{3}{8} - \left(-1\frac{1}{4}\right)$
20. $5\frac{5}{6} - \left(-2\frac{1}{3}\right)$
21. $-18.7 - (-9.3)$
22. $107.4 - (-12.6)$
23. $-215.8 - 96.2$
24. $-119.1 - 218.8$
25. $-512.7 - (-814.5)$
26. $(-12) - (-10) - (8)$
27. $(-30) + (14) - (8)$
28. $(-25) + (4) - (32) + (28)$
29. $(24) - (-12) - (12) + (-13)$
30. $(-2) - (3) + (-4) - (-5) + (-6)$
31. $(-15) - (13) - (-7) - (32)$
32. $(-17) - (11) - (-12) - (-5)$

Find each sum or difference. See example 1–5 B.

Example $14 - 11 + 18 - (7 - 12) + 2$

Solution
$$\begin{aligned} &= 14 - 11 + 18 - (-5) + 2 \\ &= 3 + 18 - (-5) + 2 \\ &= 21 - (-5) + 2 \\ &= 26 + 2 \\ &= 28 \end{aligned}$$

Perform operations within parentheses first, then add and subtract from left to right

33. $17 + 4 - (7 - 2)$
34. $(25 - 2) - (12 - 3)$
35. $(-6) - 4 + 8 - (8 - 7)$
36. $32 - 5 + 7 - 4 - (11 - 8)$
37. $10 - 10 + (10 + 10) - 10$
38. $12 + 3 - 16 - 10 - (12 + 5)$
39. $10 + (2 - 21) - (7 - 8)$
40. $(12 + 3) - 16 - 10 + (12 - 5)$
41. $(18 - 14) - (12 - 17) - 16$
42. $8 - 4 + 7 - (5 - 2) - 3$

See example 1–5 C.

Example A temperature of 14° C (Celsius) is decreased by 18 degrees Celsius. What is the resulting temperature?

Solution $14 - 18 = 14 + (-18)$
$= -4° \text{ C}$

43. A temperature of $(-6)°$ C is decreased by 32° C. What is the resulting temperature?
44. An electronics supply house has 432 resistors of a certain type. If 36 are sold during the first week, 72 during the second week, 29 during the third week, and 58 during the fourth week, how many are left at the end of the month?
45. Tim owes Tom and Rob \$343 and \$205, respectively, and Terry owes Tim \$176. In terms of positive and negative symbols, how does Tim stand monetarily?
46. If a person has \$78 after paying off a debt of \$23, how much money did he have before paying off the debt? Write a statement involving the operation of subtraction of integers to show your answer.
47. A piece of wood 24 feet long is cut into three pieces so that two of the pieces measure 8 feet and 10 feet. What is the length of the third piece?

Find the difference.

Example 12 diminished by 20

Solution $12 - 20$ Diminished by 20 means subtract 20
$= 12 + (-20)$ Change to addition
$= -8$

48. -8 diminished by 11
49. -15 diminished by 7
50. -6 diminished by -21
51. -18 diminished by -9
52. Subtract -26 from -18.
53. Subtract -19 from 41.
54. Subtract -17 from 28.
55. From -26 subtract -45.
56. From -43 subtract -16.
57. 5 less than -8
58. 4 less than -12
59. 8 less than 3
60. 10 less than 5

Choose a letter for the unknown quantity and find the indicated difference. See example 1–5 C.

61. Rob has \$23 in his savings account. If he spends \$15 to buy a game, how much is left in his savings account?
62. Tom has \$44 in his savings account. He wishes to buy a baseball glove for \$21. How much is left in his savings account?
63. The temperature was $-15°$ at 6 A.M. but by noon the temperature has risen to 23°. How many degrees did it rise from 6 A.M. to noon?
64. The temperature dropped 22° from $-7°$ at midnight to just before daybreak at 7 A.M. What was the temperature at 7 A.M.?
65. Erin Nustad was born in 1986. How old will she be in the year 2000?
66. A chemist has 100 ml of acid and she needs 368 ml of the acid. How much more is needed?
67. Amy has \$450 in assets and she wants to borrow enough money to buy a stereo system for \$695. How much will she owe?
68. The top of Mt. Everest in Asia is 29,028 feet above sea level and the top of Mt. McKinley in Alaska is 20,320 feet above sea level. How much higher is the top of Mt. Everest than the top of Mt. McKinley?
69. Death Valley in California is 282 feet below sea level (-282). What is the difference in the altitude between Mt. McKinley and Death Valley? See exercise 68.
70. Mt. Whitney in California is 14,494 feet above sea level and the Salton Sea in California is 235 feet below sea level. What is the difference between the altitude of Mt. Whitney and Salton Sea?

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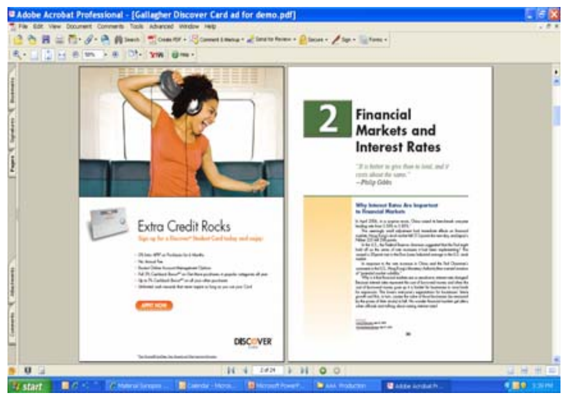

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71. Jim owes John \$25. He paid back \$13 and then had to borrow another \$7. How much does Jim owe John now?

72. On 4 successive hands in a poker game, Sheila won \$10, then lost \$9, then lost \$14, and finally lost \$6. What is her financial position after the 4 hands?

1–6 ■ Multiplication of real numbers

Multiplication of two positive numbers

We are already familiar with the fact that the product of two positive numbers is positive. We can see this fact by considering multiplication as repeated addition. For example, if we wish to add four 3s, then $3 + 3 + 3 + 3 = 12$. Another way of expressing this repeated addition is $4 \cdot 3 = 12$, in which case the raised dot, $\cdot$, means multiply or times. We could also have added three 4s: $4 + 4 + 4 = 12$, which could be written as $3 \cdot 4 = 12$. This observation illustrates an important mathematics principle called the **commutative property of multiplication.**

Commutative property of multiplication

For every real number a and b,

$$a \cdot b = b \cdot a$$

Concept

This property tells us that changing the order of the numbers when we multiply will not change the answer (product).

In the previous paragraph, the number 12 is called the **product** of 4 and 3, and 4 and 3 are called **factors** of 12. *The numbers or variables in an indicated multiplication are referred to as the factors of the product.*

In our example, we used a raised dot to indicate the operation of multiplication. The cross, $\times$, is used in arithmetic to indicate multiplication. We avoid using it in algebra because it may become confused with the variable x. Another way to indicate multiplication is the absence of any operation symbol between factors. The following are other examples of how we can express multiplication.

■ **Example 1–6 A**

1. $5 \cdot 7$ is read 5 times 7. The raised dot indicates multiplication.
2. $(4)(6)$ is read 4 times 6. The parentheses separate the numbers; the absence of any operation symbol between the numbers indicates multiplication.
3. $3a$ is read 3 times a.
4. ab is read a times b.
5. $6(8)$ is read 6 times 8. ■

Note 34 does not mean $3 \cdot 4$ and $3\frac{1}{2}$ does not mean $3 \cdot \frac{1}{2}$

Multiplication of two numbers with different signs

As an illustration of multiplying a positive number times a negative number, consider the following pattern:

$$\begin{aligned} 3 \cdot 3 &= 9 \\ 2 \cdot 3 &= 6 \\ 1 \cdot 3 &= 3 \\ 0 \cdot 3 &= 0 \\ (-1) \cdot 3 &= -3 \\ (-2) \cdot 3 &= -6 \\ (-3) \cdot 3 &= -9 \end{aligned}$$

The product decreases by 3

The product of a negative and a positive is a negative

We observe from this pattern that our product decreases by 3 each time. It logically follows that the product of a negative number and a positive number is a negative number.

A second observation from this pattern is that zero times a number has a product of zero. This is called the **zero factor property.**

Zero factor property

For every real number a,

$$a \cdot 0 = 0 \cdot a = 0$$

Concept

Multiplying any number by zero always gives zero as the answer. That is, whenever we are multiplying and zero is one of the factors, our product will be zero.

A third observation from this pattern is that 1 times a number is equal to the number. For this reason, 1 is called the **identity element of multiplication.**

Identity property of multiplication

For every real number a

$$a \cdot 1 = 1 \cdot a = a$$

Concept

Multiplying a number by 1 leaves the number unchanged.

Multiplication of two negative numbers

We will observe another pattern when we consider the following.

$$\begin{aligned} 3(-3) &= -9 \\ 2(-3) &= -6 \\ 1(-3) &= -3 \\ 0(-3) &= 0 \\ (-1)(-3) &= +3 \\ (-2)(-3) &= +6 \\ (-3)(-3) &= +9 \end{aligned}$$

The product increases by 3

The product of two negatives is a positive

From this pattern, we can see that our product increases by 3 each time. It logically follows that *the product of two negative numbers is a positive number.*

We can summarize our procedures for multiplication of real numbers as follows:

Multiplication of two real numbers

To multiply two real numbers, multiply their absolute values and
1. the product will be positive if the numbers have the same sign;
2. the product will be negative if the numbers have different signs.

■ Example 1–6 B

Multiply the following numbers.

1. $(-2) \cdot 3 = -6$

Product of their absolute values: $2 \cdot 3 = 6$
Product is negative because the numbers have different signs

2. $(-2)(-4) = 8$

Product of their absolute values: $2 \cdot 4 = 8$
Product is positive because the numbers have the same signs

3. $4(-4) = -16$ Negative because signs are different

4. $(-5)(-5) = 25$ Positive because signs are the same

▶ ***Quick check*** $(-4)(-3)$ ■

We can determine the sign of our answer when we multiply three or more real numbers. Consider the following examples.

1. $(-1)(2)(3)(4) = (-2)(3)(4) = (-6)(4) = -24$ Odd number of negative factors
2. $(-1)(2)(-3)(4) = (-2)(-3)(4) = (6)(4) = 24$ Even number of negative factors
3. $(-1)(2)(-3)(-4) = (-2)(-3)(-4) = (6)(-4) = -24$ Odd number of negative factors
4. $(-1)(-2)(-3)(-4) = (2)(-3)(-4) = (-6)(-4) = 24$ Even number of negative factors

Multiplication of two or more real numbers

1. If in the numbers being multiplied there is an **odd** number of negative factors, the answer will be negative.
2. If in the numbers being multiplied there is an **even** number of negative factors, the answer will be positive.

■ Example 1–6 C

Multiply the following numbers.

1. $(-7)(-2)(5) = (14)(5) = 70$ Even number of negative factors

2. $(-6)(2)(-4) = (-12)(-4) = 48$ Even number of negative factors

3. $[(-3)(5)](4) = (-15)(4) = -60$ Odd number of negative factors

4. $(-3)[(5)(4)] = (-3)(20) = -60$ Odd number of negative factors

▶ ***Quick check*** $(-3)(-2) \cdot 4$ ■

Examples 3 and 4 illustrate an important mathematical principle called the **associative property of multiplication.**

Associative property of multiplication

For every real number a, b, and c,

$$(a \cdot b)c = a(b \cdot c)$$

Concept

Changing the grouping of the numbers will not change the product.

Problem solving

To solve the following problems, we must multiply the quantities.

■ Example 1–6 D

Choose a letter for the unknown and find the indicated product.

1. What is the cost of 7 VHS tapes if each tape costs \$5.95?

   Let c = the cost of the 7 tapes. We must multiply to find the total cost of all 7 tapes. Thus

total cost	is	7 tapes	at	\$5.95 each
c	$=$	7	$\cdot$	(5.95)

   $c = 7 \cdot (5.95) = 41.65$

   The 7 tapes cost \$41.65.

2. On 4 successive days, the stock market dropped 9 points (represented by a negative number) each day. How many points did the market change in the 4 days?

   Let d = the total drop. Represent 9-point drop by -9. We multiply to obtain

total drop	is	4 days	at	9-point drop each day
d	$=$	4	$\cdot$	(-9)

   $d = 4 \cdot (-9) = -36$

   The stock market dropped 36 points (-36) in the 4 days. ■

Mastery points

Can you

- Use the commutative and associative properties of multiplication, the zero factor property, and the identity property of multiplication?
- Multiply real numbers?

Exercise 1–6

Perform the indicated operations. See examples 1–6 B and C.

Examples $(-4)(-3)$

Solutions $= 12$ Product is positive because the numbers have like signs

$(-3)(-2) \cdot 4$

$= 6 \cdot 4$ $(-3)(-2) = 6$ because the numbers have like signs

$= 24$ 6 times 4 is 24

1. $(-3)(-5)$
2. $0 \cdot (-6)$
3. $4 \cdot (-7)$
4. $(-8) \cdot 3$
5. $4 \cdot (-3) \cdot 5$
6. $(-2)(2)(-2)$
7. $4 \cdot (-9)$
8. $(-3)(-2)(-8)$
9. $(-1)(-4)(5)$
10. $(-5)(2)(4)(3)$
11. $7 \cdot (-1)(-3)(-5)$
12. $2 \cdot (-3)(-1)(2)(-2)(3)$
13. $(-1.8)(2.4)$
14. $(-5.7)(-6.12)$
15. $(0.49)(-28.1)$
16. $(-8.9)(-8.9)$
17. $(-27)(0.08)$
18. $\left(-\frac{1}{3}\right)\left(\frac{3}{5}\right)$
19. $\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)$
20. $\left(-\frac{3}{4}\right)\left(\frac{8}{9}\right)$
21. $\left(-\frac{5}{8}\right)\left(-\frac{2}{5}\right)$
22. $\left(\frac{5}{12}\right)\left(-\frac{9}{10}\right)$
23. $(-5)(-4)(-3)(2)$
24. $(-2)(-7)(7)(4)$
25. $(-3)(3)(-4)(4)$
26. $(-1)(-1)(-1)(-1)$
27. $(-2)(0)(3)(-4)$
28. $(-3)(-2)(4)(0)$
29. $(-5)(0)(-4)$

In exercises 30–46, two numbers are listed. Find two integers such that their product is the first number and their sum is the second number.

Examples $4, -4$

Solutions Since $(-2)(-2) = 4$ and $(-2) + (-2) = -4$, then -2 and -2 are the integers.

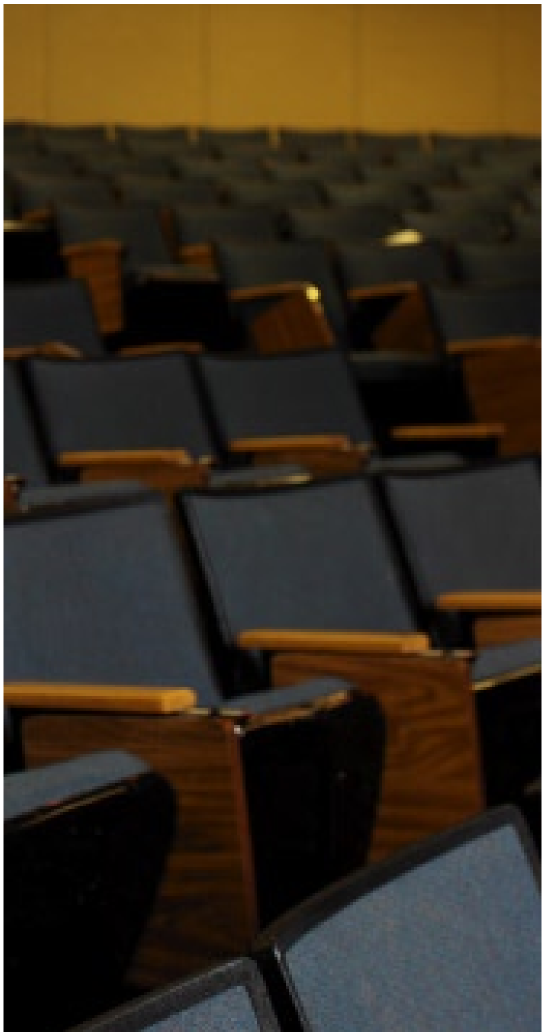
$-27, -6$

Since $(-9)(3) = -27$ and $(-9) + (3) = -6$, then -9 and 3 are the integers.

30. $-16, 0$
31. $-30, 1$
32. $25, 10$
33. $20, -9$
34. $-11, 10$
35. $0, -7$
36. $-72, -21$
37. $-12, -1$
38. $48, 16$
39. $35, 12$
40. $4, -4$
41. $-8, 7$
42. $-9, 0$
43. $-12, 1$
44. $-15, 2$
45. $-18, 3$
46. $-30, -1$

Choose a letter for the unknown and multiply to find the value. See example 1–6 D.

47. Over a five-day period, the price of a particular stock suffered losses of \$3 on each of the first two days and \$2 on each of the last three days. If the stock originally sold for \$88, what was its price after the five-day period?
48. A man acquires a debt of \$6 each day for five days. If we represent a \$6 debt by (-6), write a statement of the change in his assets after five days. What is the change?
49. An auditorium contains 42 rows of seats. If each row contains 25 seats, how many people can be seated in the auditorium?
50. There are 7 rows of desks in a classroom. If each row contains 8 desks, how many students will the classroom hold?
51. A clothier ordered 15 suits, each costing him \$65. What was the total cost of the 15 suits?



52. Mrs. Jones purchased two dozen (24) cans of frozen orange juice concentrate that was on sale for 57¢ per can. How much did the orange juice cost her?

53. Jim Johnson lost an average of \$23 in 4 successive poker games. What were his total losses? (Represent this by a negative answer.)

54. If a bank advertises that a person can double his investment in a savings account in 9 years, how much will \$120 grow to in 9 years?

55. Joanie sold 35 glasses of lemonade at her corner stand. If she charged 15¢ a glass, how much did she make in sales?

56. Berkeley Gossett deposits his weekly allowance of \$3 in a savings account at the bank. How much has he deposited in 3 years? (*Hint:* 52 weeks = 1 year.)

$\boxed{57.}$ A grocer averages selling 25 gallons of milk each day. How many gallons of milk does he sell in 4 weeks? (Assume the grocery is open 7 days per week.)

58. A department store averages a loss of \$75, due to thefts, each day. How much is lost in a month of 31 days?

1–7 ■ Division of real numbers

Division of two numbers

Recall that when we divide a number (called the *dividend*) by another number (called the *divisor*), we compute an answer (called the *quotient*). We define the operation of division as follows:

Definition of division

If $b \neq 0$*, $\frac{a}{b} = q$ provided that $b \cdot q = a$, where a is the dividend, b is the divisor, and q is the quotient.

The second part of our definition of division shows how to check the answer to the problem. We multiply the divisor by the quotient to get the dividend ($b \cdot q = a$).

By the above definition, the quotient of the two negative numbers $(-20) \div (-5)$ or $\frac{-20}{-5}$ must be that number which multiplied by -5 gives -20. That number is 4, since $(-5)(4) = -20$. Therefore $\frac{-20}{-5} = 4$. We observe that *the quotient of two negative numbers is a positive number.*

To divide a positive number by a negative number, or a negative number by a positive number, consider the following divisions:

$$(-14) \div (2) = \frac{-14}{2} = -7$$

since

$$(2)(-7) = -14$$

and

$$(24) \div (-6) = \frac{24}{-6} = -4$$

because

$$(-6)(-4) = 24$$

We find that *the quotient of a positive number and a negative number is always a negative number.*

*The reason for this restriction will be explained on page 53.

We can summarize our procedures for multiplication and division of real numbers as follows:

Multiplication or division of two real numbers

To multiply or divide two real numbers, perform the operation (multiplication or division) using the absolute values of the numbers and

1. the quotient will be positive if the numbers have like signs;
2. the quotient will be negative if the numbers have different signs.

Example 1–7 A

Divide the following numbers.

1. $\dfrac{-14}{-7} = 2$, since $(-7)(2) = -14$
2. $\dfrac{-36}{-6} = 6$, since $(-6)(6) = -36$
3. $\dfrac{-24}{3} = -8$, since $(3)(-8) = -24$
4. $\dfrac{15}{-5} = -3$, since $(-5)(-3) = 15$

▶ **Quick check** $\dfrac{-18}{-9}$; $\dfrac{-15}{3}$

In section 1–6, we developed a procedure for determining the sign of our answer when we multiply three or more real numbers. This same rule can be extended to apply to division. Consider the following examples.

Example 1–7 B

1. $\dfrac{(-1)(12)}{(2)(3)} = \dfrac{-12}{6} = -2$ Odd number of negative factors
2. $\dfrac{(-1)(12)}{(2)(-3)} = \dfrac{-12}{-6} = 2$ Even number of negative factors
3. $\dfrac{(-1)(12)}{(-2)(-3)} = \dfrac{-12}{6} = -2$ Odd number of negative factors
4. $\dfrac{(-1)(-12)}{(-2)(-3)} = \dfrac{12}{6} = 2$ Even number of negative factors

We can summarize the procedure for three or more real numbers in a multiplication or division problem as follows:

Multiplication or division of two or more real numbers

When we multiply or divide, if we have an odd number of negative factors, our answer will be negative; otherwise it will be positive.

Note Our procedure concerning multiplication or division of three or more signed numbers applies *only* when we are doing *strictly* the operations of multiplication and division. For example, in the problem $\frac{(-8) + (-4)}{-2}$, we have an odd number of negative numbers, but our solution would be as follows: $\frac{(-8) + (-4)}{-2} = \frac{-12}{-2} = 6$. We are not able to apply our procedure here because we are not performing strictly multiplication and division.

Division involving zero

In section 1–3, we defined a rational number to be any number that can be expressed as a quotient of two integers in which the divisor is not zero. The number zero, 0, is the only number that we cannot use as a divisor. To see why we exclude zero as a divisor, recall that we check a division problem by multiplying the divisor times the quotient to get the dividend. If we apply this idea in connection with zero as a divisor, we observe the following situations. Suppose there were a number q such that $3 \div 0 = q$. Then $q \cdot 0$ would have to be equal to 3 for our answer to check, but this product is zero regardless of the value of q. Therefore we cannot find an answer for this problem. We say that the answer is *undefined.* If we try to divide zero by zero and again call our answer q, we have $0 \div 0 = q$. When we check our work, $0 \cdot q = 0$, we see that any value for q will work. Since any value for q will work, we say our answer is *indeterminate.* We therefore decide that **division by zero is not allowed.**

It is important to note that although division by zero is not allowed, this does not extend to the division of zero by some other number. We can see that $\frac{0}{-4} = 0$ since $(-4) \cdot 0 = 0$. Thus, **the quotient of zero divided by any number other than zero is always zero.**

Example 1–7 C

Perform the division, if possible.

1. $\frac{0}{5} = 0$ **2.** $\frac{2}{0}$ is undefined **3.** $\frac{-7}{0}$ is undefined

4. $\frac{0}{-7} = 0$ **5.** $\frac{0}{0}$ is indeterminate

▶ ***Quick check*** Divide $\frac{11}{0}$, if possible.

Problem solving

To solve the following problems, we will have to divide the given quantities.

Example 1–7 D

Choose a letter for the unknown quantity and find the indicated quotient.

1. If \$7.68 is spent on 6 three-way light bulbs, how much did each light bulb cost?

   Let c = the cost of each light bulb. We must divide \$7.68 by 6. Thus

cost per bulb	is equal to	total cost	divided	by 6 identical items
c	$=$	(7.68)	$\div$	6

   $c = (7.68) \div 6 = 1.28$

   Each light bulb cost \$1.28.

2. There are 400 people seated in a full auditorium. If there are 25 identical rows of seats, how many people are there in each row?

   Let x = the number of people in each row. We must divide 400 by 25. Thus

people per row	is equal to	total number of people	divided	by 25 identical rows
x	$=$	400	$\div$	25

   $x = 400 \div 25 = 16$

   There are 16 people seated in each row. ■

Mastery points

Can you

- Perform division with real numbers?
- Remember the results of division involving zero?

Exercise 1–7

Perform the indicated operations, if possible. See examples 1–7 A, B, and C.

Examples	$\dfrac{-18}{-9}$	$\dfrac{-15}{3}$	$\dfrac{11}{0}$
Solutions	$= 2$ Quotient of two negatives is a positive	$= -5$ Quotient of a positive and a negative is a negative	is undefined

1. $\dfrac{-14}{-7}$ 2. $\dfrac{-15}{5}$ 3. $\dfrac{32}{-4}$ 4. $\dfrac{18}{3}$ 5. $\dfrac{-22}{-11}$

6. $\dfrac{18}{-3}$ 7. $\dfrac{-16}{2}$ 8. $\dfrac{-25}{-5}$ 9. $\dfrac{7}{0}$ 10. $\dfrac{-4}{0}$

$\boxed{11.}$ $\dfrac{0}{-9}$ 12. $\dfrac{0}{5}$ 13. $\dfrac{0}{0}$ 14. $\dfrac{-24}{-6}$ 15. $\dfrac{49}{-7}$

16. $\dfrac{36}{-6}$ 17. $\dfrac{-25}{5}$ 18. $\dfrac{-64}{8}$ $\boxed{19.}$ $\dfrac{(-4)(-3)}{-6}$ 20. $\dfrac{(-18)(2)}{-4}$

21. $\frac{(16)(2)}{-8}$ 22. $\frac{(-4)(0)}{-8}$ 23. $\frac{(-16)(0)}{-8}$ 24. $\frac{(-5)(-2)}{(-1)(-10)}$ 25. $\frac{(-18)(3)}{(-2)(-9)}$

26. $\frac{(-2)(-4)}{(0)(4)}$ 27. $\frac{(-3)(6)}{(0)(-2)}$ 28. $\frac{8-8}{3+4}$ 29. $\frac{(-6)(0)}{(-3)(0)}$ 30. $\frac{6-6}{6-6}$

Example A football player carried the ball eight times, making the following yardages: gain of 6 yards (yd), loss of 3 yd, loss of 4 yd, gain of 4 yd, gain of 3 yd, loss of 1 yd, loss of 2 yd, gain of 5 yd. Show his gains and losses by positive and negative integers. What was his average gain or loss per carry?

Solution 6 yd, −3 yd, −4 yd, 4 yd, 3 yd, −1 yd, −2 yd, 5 yd. To find an average, we add together all of the values and divide by the total number of values.

$$\frac{6\text{ yd} + (-3)\text{ yd} + (-4)\text{ yd} + 4\text{ yd} + 3\text{ yd} + (-1)\text{ yd} + (-2)\text{ yd} + 5\text{ yd}}{8} = \frac{8\text{ yd}}{8} = 1\text{ yd}$$

31. The temperature at 1 P.M. for seven consecutive days in January was 5° C, −8° C, −7° C, −1° C, 10° C, −6° C, and 0° C. What was the average temperature for the seven days?

32. If the stock market showed the following gains and losses during six consecutive hours of trading on a given day, determine the average gain or loss during that six-hour period. Gain 36 points, loss 23 points, loss 72 points, gain 25 points, loss 31 points, loss 21 points.

33. Between Chicago and Detroit, a distance of 282 miles, a driver averages 47 miles per hour. How long will it take her to make the trip?

34. A trip of 369 miles takes nine hours to complete. What was the average rate of speed?

35. Light travels at a rate of 186,000 miles per second. How long will it take to travel 1,674,000 miles?

36. How long does it take light from the sun to reach earth if the sun is approximately 93,000,000 miles away? (Refer to exercise 35.)

Choose a letter for the unknown quantity and find the value by dividing. See example 1–7 D.

37. Mrs. Smith paid \$36 for 9 crates of peaches for her fruit market. How much did each crate cost her?

38. A carpenter wishes to cut a 12-foot board into 3 pieces that are all the same length. Find the length of each piece.

39. A man drove 350 miles and used 14 gallons of gasoline. How many miles did he drive on each gallon of gasoline?

40. Irene drove 424 miles in 8 hours. How many miles did she travel each hour (in miles per hour) if she drove at a constant speed?

41. During a recent cold wave, the temperature fell 28° over a 7-day period. What was the average change per day?

42. Mary Ann typed 1,350 words in 30 minutes. How many words did she type per minute?

43. The college bookstore purchased 480 math textbooks. If the books came in 15 boxes of the same size, how many books were in each box?

44. Jim, John, Pete, and Mike worked together painting farmer Gene's barn. If he gave them \$124 to split evenly among them, how much did each boy receive?

45. Alice took part in a 26-mile marathon run. If she ran the marathon in 5 hours and 12 minutes, how long did it take her to run 1 mile (in minutes) if she ran at a constant speed?

46. A farmer got 720 bushels of wheat from a 30-acre field. How many bushels did he get per acre?

1–8 ■ Properties of real numbers and order of operations

In the previous four sections, we introduced some of the properties of real numbers. We also saw how these properties are used when performing fundamental operations with numbers. Since variables represent numbers, we will be using these and other properties throughout our study of algebra. The properties that we have covered so far are listed and the page number where the property was first introduced is given for reference.

Properties of real numbers

If a, b, and c are any real numbers, then

$a + b = b + a$, commutative property of addition (page 35)
$a \cdot b = b \cdot a$, commutative property of multiplication (page 46)
$(a + b) + c = a + (b + c)$, associative property of addition (page 38)
$(a \cdot b)c = a(b \cdot c)$, associative property of multiplication (page 49)
$a \cdot 1 = 1 \cdot a = a$, identity property of multiplication (page 47)
$a + 0 = 0 + a = a$, identity property of addition (page 35)
$a + (-a) = 0$, additive inverse property (page 38)
$a \cdot 0 = 0 \cdot a = 0$, zero factor property (page 47)

Exponents

Consider the indicated products

$$4 \cdot 4 \cdot 4 = 64$$

and

$$3 \cdot 3 \cdot 3 \cdot 3 = 81$$

A more convenient way of writing $4 \cdot 4 \cdot 4$ is 4^3, which is read "4 to the third power" or "4 cubed." We call the number 4 the **base** of the expression and the number 3, to the upper right of 4, the **exponent.**

Thus

Exponent

Exponential form → $4^3 = 4 \cdot 4 \cdot 4 = 64$ ← Standard form

Base

Expanded form

In like fashion, $3 \cdot 3 \cdot 3 \cdot 3$ may be written 3^4, where 3 is the base and 4 is the exponent. The expression is read "3 to the fourth power." Then

$$3 \cdot 3 \cdot 3 \cdot 3 = 3^4 = 81$$

Notice that *the exponent tells how many times the base is used as a factor in an indicated product.* We call this form of a product the **exponential form.** That is, the exponential form of the product $3 \cdot 3 \cdot 3 \cdot 3$ is 3^4.

Note The exponent is understood to be 1 when a number has no exponent. That is, $5 = 5^1$.

Remember that when we have a negative number, we place it inside parentheses. With this fact in mind, we can see that there is a definite difference between $(-2)^4$, which is read "-2 to the fourth power," and -2^4, which is read "the opposite of 2 to the fourth power." In the first case, the parentheses denote that this is a negative number to a power: $(-2)^4 = (-2)(-2)(-2)(-2) = +16$. In the second case, since there are no parentheses around the number, we understand that this is *not* (-2) to a power. It is, rather, the opposite of the answer when we raise 2^4: $-2^4 = -(2)^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -(16)$.

Example 1–8 A

Perform the indicated multiplication.

1. $(-3)^3 = (-3)(-3)(-3) = -27$
2. $-3^3 = -(3 \cdot 3 \cdot 3) = -27$
3. $(-3)^4 = (-3)(-3)(-3)(-3) = 81$
4. $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

▶ ***Quick check*** -3^2

When we are performing several different types of arithmetic operations within an expression, we need to agree on an order in which the operations will be performed. To show that this is necessary, consider the following numerical expression.

$$3 + 4 \cdot 5 - 3$$

More than one answer is possible, depending on the order in which we perform the operations. To illustrate,

$$3 + 4 \cdot 5 - 3 = 7 \cdot 2 = 14$$

if we add and subtract as indicated before we multiply. However

$$3 + 4 \cdot 5 - 3 = 3 + 20 - 3 = 20^*$$

if we multiply before we add or subtract. A third possibility would be

$$3 + 4 \cdot 5 - 3 = 3 + 4 \cdot 2 = 3 + 8 = 11$$

if we subtract, then multiply, and finally add. To standardize the answer, we agree to the following order of operations, or priorities.

Order of operations, or priorities

1. **Groups:** Perform any operations within a grouping symbol such as ( ) parentheses, [ ] brackets, { } braces, | | absolute value, and above or below the fraction bar.
2. **Exponents:** Perform operations indicated by exponents.
3. **Multiply and divide:** Perform multiplication and division in order from left to right.
4. **Add and subtract:** Perform addition and subtraction in order from left to right.

Note

a. Within a grouping symbol, the order of operations will still apply.
b. If there are several grouping symbols intermixed, remove them by starting with the innermost one and working outward.

*This is the correct answer.

To illustrate this order, consider the numerical expression

$$6 + 5(7 - 3) - 2^2$$

We first evaluate within the grouping symbol, in this case parentheses, to get

$$6 + 5(4) - 2^2$$

We then perform the indicated power and have

$$6 + 5(4) - 4$$

Our third step is to carry out the multiplication, resulting in

$$6 + 20 - 4$$

Our last step is to perform the addition and subtraction in order from left to right, giving

$$\begin{aligned} &26 - 4 \\ &= 22 \end{aligned}$$

■ **Example 1–8 B**

Perform the indicated operations in the proper order and simplify.

1. $$\begin{aligned} 7 + 8 \cdot 3 \div 2 &= 7 + 24 \div 2 && \text{Priority 3, multiply} \\ &= 7 + 12 && \text{Priority 3, divide} \\ &= 19 && \text{Priority 4, add} \end{aligned}$$

2. $$\begin{aligned} (7 - 1) \div 2 + 3 \cdot 4 &= 6 \div 2 + 3 \cdot 4 && \text{Priority 1, parentheses} \\ &= 3 + 12 && \text{Priority 3, divide and multiply} \\ &= 15 && \text{Priority 4, add} \end{aligned}$$

3. $$\begin{aligned} \frac{1}{2} + \frac{3}{4} \div \frac{5}{8} &= \frac{1}{2} + \frac{3}{\cancelto{1}{4}} \cdot \frac{\cancelto{2}{8}}{5} && \text{Priority 3, invert, divide out common factors} \\ &= \frac{1}{2} + \frac{6}{5} && \text{Priority 3, multiply} \\ &= \frac{5}{10} + \frac{12}{10} && \text{Least common denominator} \\ &= \frac{5 + 12}{10} && \text{Priority 4, add} \\ &= \frac{17}{10} \text{ or } 1\frac{7}{10} && \text{Priority 4, add} \end{aligned}$$

4. $$\begin{aligned} 2^2 \cdot 3 - 3 \cdot 4 &= 4 \cdot 3 - 3 \cdot 4 && \text{Priority 2, exponent} \\ &= 12 - 12 && \text{Priority 3, multiply} \\ &= 0 && \text{Priority 4, subtract} \end{aligned}$$

5. $$\begin{aligned} \frac{3}{4} - \frac{1}{2} \cdot \frac{2}{3} &= \frac{3}{4} - \frac{1}{\cancelto{1}{2}} \cdot \frac{\cancelto{1}{2}}{3} && \text{Priority 3, divide out common factors} \\ &= \frac{3}{4} - \frac{1}{3} && \text{Priority 3, multiply} \\ &= \frac{9}{12} - \frac{4}{12} && \text{Least common denominator} \\ &= \frac{9 - 4}{12} && \text{Priority 4, subtract} \\ &= \frac{5}{12} && \text{Priority 4, subtract} \end{aligned}$$

6. $(7.28 + 1.6) \div 2.4 - (6.1)(3.8)$

$= (8.88) \div 2.4 - (6.1)(3.8)$ Priority 1, parentheses

$= 3.7 - 23.18$ Priority 3, division and multiplication

$= -19.48$ Priority 4, subtract

7. $\left(\frac{2}{3} + \frac{7}{8}\right) \div \frac{5}{6} = \left(\frac{16}{24} + \frac{21}{24}\right) \div \frac{5}{6}$ Priority 1, parentheses

$= \left(\frac{16 + 21}{24}\right) \div \frac{5}{6}$ Priority 1, parentheses

$= \frac{37}{24} \div \frac{5}{6}$ Priority 1, parentheses

$= \frac{37}{\cancel{24}_{4}} \cdot \frac{\cancel{6}^{1}}{5}$ Priority 3, invert, divide out common factors

$= \frac{37}{20}$ or $1\frac{17}{20}$ Priority 3, multiply

8. $(5.4)^2 - 4(3.1)(2.8)$

$= 29.16 - 4(3.1)(2.8)$ Priority 2, exponent

$= 29.16 - 34.72$ Priority 3, multiply

$= -5.56$ Priority 4, subtract

9. $\frac{3(2 + 4)}{4 - 2} - \frac{4 + 6}{5} = \frac{3(6)}{4 - 2} - \frac{4 + 6}{5}$ Priority 1, groups: numerator and denominator

$= \frac{18}{2} - \frac{10}{5}$ Priority 1, numerator and denominator

$= 9 - 2$ Priority 3, divide

$= 7$ Priority 4, subtract

10. $5[7 + 3(10 - 4)]$

We first evaluate within the grouping symbol, applying the order of operations.

$5[7 + 3(6)] = 5[7 + 18]$ Priority 1, groups

$= 5[25]$ Priority 1, groups

$= 125$ Priority 3, multiply

▶ ***Quick check*** $18 \div 6 \cdot 3 + 10 - (4 + 5)$ ■

Problem solving

Solve the following word problems using the order of operations.

■ ***Example 1–8 C***

Choose a variable to represent the unknown quantity and find its value by performing the indicated operations.

1. Mrs. Hansen purchased 6 boxes of cereal at \$1.25 per box and 7 cans of tuna fish at 70¢ per can. What was her total bill?

Let t = Mrs. Hansen's total bill. 6 boxes at \$1.25 per box cost $6 \cdot \$1.25$; 7 cans at 70¢ per can cost $7 \cdot \$0.70$. The total bill is given by

total bill	is equal to	6 boxes of cereal	at	\$1.25 per box	and	7 cans of tuna	at	\$0.70 per can
t	$=$	6	$\cdot$	(1.25)	$+$	7	$\cdot$	(0.70)

$t = 6 \cdot (1.25) + 7 \cdot (0.70)$
$= 7.50 + 4.90$ Priority 3
$= 12.40$ Priority 4

Mrs. Hansen's total bill was \$12.40.

2. A man works a 40-hour week at \$12 per hour. If he works 11 hours of overtime at time and a half, how much will he receive for the 51 hours of work?

Let w = the man's total wages for the week. 40 hours at \$12 per hour is $40 \cdot \$12$. Hourly rate at time and a half is $\left(1\frac{1}{2} \cdot 12 = 18\right)$ and 11 hours at time and a half is $11 \cdot 18$. Thus

total wages	is equal to	40 hours	at	\$12 per hour	and	11 hours	at	\$18 per hour
w	$=$	40	$\cdot$	12	$+$	11	$\cdot$	18

$w = 40 \cdot 12 + 11 \cdot 18$
$w = 480 + 198$ Priority 3
$w = 678$ Priority 4

The man will receive \$678 for 51 hours of work. ■

Mastery points

Can you

- Perform multiple operations in the proper order?
- Use exponents?

Exercise 1–8

Perform the indicated operations. See example 1–8 A.

Example -3^2

Solution $= -(3^2)$ -3^2 is not the same as $(-3)^2$, it is the opposite of 3^2
$= -9$

1. $(-4)^2$ $\boxed{2.}$ $(-5)^4$ $\boxed{3.}$ $(-3)^3$ **4.** -4^2 **5.** -6^2

6. -2^4 **7.** -1^2 $\boxed{8.}$ -2^2 **9.** $(-1)^2$ **10.** $(-2)^2$

Perform the indicated operations and simplify. See example 1–8 B.

Example $18 \div 6 \cdot 3 + 10 - (4 + 5)$

Solution

$= 18 \div 6 \cdot 3 + 10 - 9$	Priority 1, parentheses
$= 3 \cdot 3 + 10 - 9$	Priority 3, division
$= 9 + 10 - 9$	Priority 3, multiplication
$= 19 - 9$	Priority 4, addition
$= 10$	Priority 4, subtraction

11. $\dfrac{4+2}{3} + 2$

12. $-6 \cdot 7 + 8$

13. $6 + 5 \cdot 4$

14. $\dfrac{1}{5} \cdot 5 + 6$

15. $-2 + 10 \cdot \dfrac{1}{5}$

16. $4(3 - 2)(2 + 1)$

17. $0(5 + 2) + 3$

18. $\dfrac{24 \cdot 3}{9} - 6$

19. $(24 - 6) \div 3$

20. $(37 - 4) \div 11$

21. $\dfrac{2}{3} \div \left(\dfrac{5}{6} - \dfrac{4}{9}\right)$

22. $12 \cdot 4 + 2$

23. $2 + 3(8 - 5)$

24. $5 + 2(11 - 6)$

25. $6 + 4(8 + 2)$

26. $7 + 3(9 - 4)$

27. $8 - 3(6 - 4)$

28. $10 - 2(7 - 11)$

29. $15 \cdot 3^2 - 14$

30. $(8 - 3)(5 + 3)$

31. $\dfrac{7}{8} - \dfrac{1}{2} \div \dfrac{3}{4}$

32. $\dfrac{3}{8} + \dfrac{7}{12} \cdot \dfrac{3}{14}$

33. $3(6 - 2)(7 + 1)$

34. $12 + 3 \cdot 16 \div 4^2 - 2$

35. $9 - 3(12 + 3) - 4 \cdot 3$

36. $15 - 2(8 + 1) - 6 \cdot 4$

37. $50 - 4(6 - 8) + 5 \cdot 4$

38. $18 - 5(7 + 3) - 6$

39. $10 - 3 \cdot 4 \div 6 - 5$

40. $8 - (12 + 3) - 4 \cdot 3$

41. $4(2 - 5)^2 - 2(3 - 4)$

42. $6(-8 + 10) - 5(4 - 7)$

43. $\dfrac{5(3-5)}{2} - \dfrac{27}{-3}$

44. $\dfrac{3(8-6)}{2} - \dfrac{8}{-2}$

45. $\dfrac{5(6-3)}{3} - \dfrac{(-14)}{2}$

46. $(14.13 + 11.4) \div 3.7 - (2.4)(7.8)$

47. $(5.1 + 2.2)(4.8) - (6.3)(8.1)$

48. $(5.1)^2 \cdot 3 - (14.64) \div (6.1)$

49. $(1.9)^2 + 4(3.3)^2 - 8.7$

50. $5[10 - 2(4 - 3) + 1]$

51. $18 + [14 - 5(6 - 4) + 7]$

52. $(8 - 2)[16 + 4(5 - 7)]$

53. $(9 - 6)[21 + 5(4 - 6)]$

54. $\left(\dfrac{6-3}{7-4}\right)\left(\dfrac{14 + 2 \cdot 3}{5}\right)$

55. $\left(\dfrac{3}{12} - \dfrac{1}{6}\right)\left(\dfrac{2}{3} + \dfrac{1}{8}\right)$

56. $\left(\dfrac{1}{4} - \dfrac{1}{6}\right) \div \left(\dfrac{2}{3} - \dfrac{1}{8}\right)$

Perform the indicated operations and simplify. See example 1–8 B.

57. To convert 74° Fahrenheit (F) to Celsius (C), we use the expression

$C = \dfrac{5}{9}(F - 32)$; thus, in this case,

$C = \dfrac{5}{9}(74 - 32)$. Find C.

58. A Murray Loop is used to determine the point at which a telephone line is grounded. The unknown distance to the point of the ground, x, for a length of the loop of 32 miles and resistances of 222 and 384 ohms is given by

$$x = \frac{384}{222 + 384} \cdot 32$$

Find x.

$\boxed{59.}$ The surface area in square inches of a flat ring whose inside radius is 2 inches and whose outside radius is 3 inches is approximately

$$\frac{22}{7} \cdot 3^2 - \frac{22}{7} \cdot 2^2$$

Find the area of this surface in square inches.

60. The surface area in square inches of a ring section whose inside diameter is 18 inches and whose outside diameter is 26 inches is approximately

$$\frac{22}{7} \cdot \frac{26 + 18}{2} \cdot \frac{26 - 18}{2}$$

Find the area of this surface in square inches.

61. To find the pitch diameter, D, of a gear with 36 teeth and an outside diameter of 8 inches, we use

$$D = \frac{36(8)}{38 + 3}$$

Find D in inches.

Choose a letter for the unknown quantity and use the order of operations to find its value. See example 1–8 C.

62. A woman purchased a case of soda (24 bottles) at 15¢ per bottle, 5 pounds of candy at 49¢ per pound, and 20 jars of baby food at 75¢ per jar. What was her total bill (a) in cents, (b) in dollars and cents?

63. Colleen Meadow is a typist in a law firm. Her base pay is \$7 per hour for a 40-hour week and she receives time and a half for every hour she works over 40 hours in a week. How much will she earn if she works 49 hours in one week?

64. In a series of poker games, Ace McGee won \$4,000 in each of 3 games, lost \$1,500 in each of 4 games, and won \$2,000 in each of 2 games. How much did Ace win (or lose) in the 9 games?

$\boxed{65.}$ A carpenter must cut a 16-foot long board into 4-foot lengths and a 12-foot long board into 3-foot lengths. How many pieces of lumber will he have?

66. Jane, David, and Mary are typists in an office. David can type 75 words per minute, Jane can type 80 words per minute, and Mary can type 95 words per minute. How many words can they type together in 15 minutes?

67. The stock market opened at 2,725 points on a given day. If it lost 9 points per hour during the first 3 hours after opening and then gained 6 points per hour during the next 5 hours, what did the stock market close at?

Chapter 1 lead-in problem

While on a trip to Canada, Tonya heard on the radio that the temperature today will be 20° Celsius. Will she need her winter coat? What will the temperature be in degrees Fahrenheit? We can determine what 20° Celsius is in degrees Fahrenheit by the following expression:

$$F = \frac{9}{5} \cdot 20 + 32$$

Solution

$F = \frac{9}{5} \cdot 20 + 32$	Original expression
$F = 36 + 32$	Order of operations: multiply
$F = 68$	Add

The temperature is 68 degrees Fahrenheit. She will not need her winter coat.

Chapter 1 summary

1. A **prime number** is any whole number greater than 1 whose only factors are the number itself and 1.
2. To **reduce a fraction** to lowest terms
   a. Write the numerator and the denominator as a product of prime factors.
   b. Divide the numerator and the denominator by the common factors.
3. To **multiply** fractions
   a. Multiply the numerators and the denominators and place the product of the numerators over the product of the denominators.
   b. Reduce the resulting fraction to lowest terms.
4. To **divide** fractions, multiply the first fraction by the reciprocal of the second fraction and reduce to lowest terms.
5. To **add** or **subtract** fractions having the same denominator, add or subtract the numerators and place this sum or difference over the common denominator.
6. The **least common denominator** (LCD) of two or more fractions is the least (smallest) number that is exactly divisible by the denominators.
7. **Percent** is defined to be parts per one hundred.
8. To change a **percent to a decimal number,** move the decimal point two places to the left and drop the % symbol.
9. To change from a **decimal number to a percent,** move the decimal point two places to the right and affix the % symbol.
10. To change a **fraction to percent,** drop the % symbol and write the number over a denominator of 100.
11. A **set** is any collection of things.
12. The set A is a **subset** of the set B, which is denoted by $A \subseteq B$, if every element in A is also an element of B.
13. We use the following *sets of numbers:*
    N, natural numbers
    W, whole numbers
    J, integers
    Q, rational numbers
    H, irrational numbers
    R, real numbers
14. A **variable** is a symbol that represents an unspecified number.
15. The **number line** is a line on which we visually represent the set of real numbers.
16. The four **inequality symbols** that denote an order relationship between numbers are $<$ (less than), $>$ (greater than), $\leq$ (less than or equal to), $\geq$ (greater than or equal to).
17. The **absolute value,** $|\quad|$, of a number is the undirected distance that the number is from the origin.
18. For any two real numbers, a and b, $a - b = a + (-b)$.
19. If any quantity is enclosed with **grouping symbols,** we treat the quantity within as a single number.
20. The numbers or variables in an indicated multiplication are referred to as the **factors** of the **product.**
21. The **exponent** tells how many times the base is used as a factor in an indicated product.
22. If $b \neq 0, \dfrac{a}{b} = q$, provided that $b \cdot q = a$.
23. *Division by zero is not allowed.*
24. The quotient of *zero* divided by any number other than zero is always zero.
25. We use the following *properties of real numbers:*
    If a, b, and c are any real numbers, then
    $a + b = b + a$, commutative property of addition
    $a \cdot b = b \cdot a$, commutative property of multiplication
    $(a + b) + c = a + (b + c)$, associative property of addition
    $(a \cdot b)c = a(b \cdot c)$, associative property of multiplication
    $a \cdot 1 = 1 \cdot a = a$, identity property for multiplication
    $a + 0 = 0 + a = a$, identity property for addition
    $a + (-a) = (-a) + a = 0$, additive inverse property
    $a \cdot 0 = 0 \cdot a = 0$, zero factor property
26. **Operations with signed numbers**

    Addition

    Same signs: Add their absolute values and prefix the sum by their common sign.

    Different signs: Subtract the lesser absolute value from the greater absolute value. The result has the sign of the number with the greater absolute value.

    Additive inverse: The sum of a number and its additive inverse (opposite) is zero.

    Subtraction

    Change the sign of the number being subtracted and add that to the first number.

    Multiplication and division

    Perform the operation (multiplication or division) using the absolute value of the numbers.

    Same signs: Answer will be positive.

    Different signs: Answer will be negative.
27. *Order of Operations*
    a. Groups: Perform any operations within a grouping symbol such as ( ) parentheses, [ ] brackets, { } braces, | | absolute value, or in the numerator or the denominator of a fraction.
    b. Exponents: Perform operation indicated by exponents.
    c. Multiplication and Division: Perform multiplication and division in order from left to right.
    d. Addition and Subtraction: Perform addition and subtraction in order from left to right.

Chapter 1 error analysis

1. Determining order between numbers
   Example: $-3 < -5$
   Correct answer: $-3 > -5$
   What error was made? (*see page 30*)
2. Evaluate absolute value
   Example: $-|-3| = 3$
   Correct answer: $-|-3| = -3$
   What error was made? (*see page 31*)
3. Adding real numbers
   Example: $(-3) + 4 = 7$
   Correct answer: $(-3) + 4 = 1$
   What error was made? (*see page 37*)
4. Subtracting real numbers
   Example: $(-9) - (-4) = -13$
   Correct answer: $(-9) - (-4) = -5$
   What error was made? (*see page 41*)
5. Combining using grouping symbols
   Example: $4 - (5 - 2) = 4 - 5 - 2 = -3$
   Correct answer: $4 - (5 - 2) = 1$
   What error was made? (*see page 42*)
6. Exponents
   Example: $-3^2 = 9$
   Correct answer: $3\frac{1}{2} = -9$
   What error was made? (*see page 57*)
7. Multiplication of negative numbers
   Example: $(-2)(-6) = -12$
   Correct answer: $(-2)(-6) = 12$
   What error was made? (*see page 48*)
8. Division of real numbers
   Example: $\frac{-15}{3} = 5$
   Correct answer: $\frac{-15}{3} = -5$
   What error was made? (*see page 52*)
9. Division by zero
   Example: $\frac{-5}{0} = 0$
   Correct answer: $\frac{-5}{0}$ is undefined.
   What error was made? (*see page 53*)
10. Exponents
    Example: $3^3 = 9$
    Correct answer: $3^3 = 27$
    What error was made? (*see page 56*)

Chapter 1 critical thinking

A watch is started at 12 noon. Each time the watch reaches the next hour, it is stopped for 10 minutes. How long will it take the watch to go from 12 noon to 12 midnight?

Chapter 1 review

[1–1]

Reduce each fraction to lowest terms.

1. $\frac{10}{14}$
2. $\frac{36}{48}$
3. $\frac{120}{180}$

Multiply or divide the following as indicated. Reduce to lowest terms.

4. $\frac{6}{7} \cdot \frac{5}{3}$
5. $\frac{2}{3} \cdot \frac{9}{10}$
6. $\frac{7}{8} \div \frac{5}{6}$
7. $\frac{5}{12} \div \frac{10}{21}$
8. $3\frac{3}{4} \div 1\frac{1}{5}$
9. $2\frac{1}{2} \cdot 3\frac{1}{3}$
10. Hannah rents $\frac{3}{4}$ of a plot of land. If the plot is $\frac{5}{6}$ of an acre in size, how many acres does Hannah rent?
11. A recipe calls for $\frac{4}{5}$ of a cup of sugar. If Dene wishes to make $\frac{1}{2}$ of the recipe, how many cups of sugar should she use?

Add or subtract the following fractions as indicated. Reduce to lowest terms.

12. $\frac{3}{7} + \frac{5}{7}$

13. $\frac{5}{8} + \frac{1}{6}$

14. $\frac{11}{12} - \frac{1}{12}$

15. $\frac{8}{9} - \frac{2}{3}$

16. $4\frac{1}{4} + 2\frac{3}{5}$

17. $\frac{1}{5} - \frac{2}{3}$

18. Paula paid $\frac{1}{3}$ of her debt one week and $\frac{1}{4}$ of her debt the second week. At the end of the second week, how much of her debt had she paid off?

19. Bob Burger owns $3\frac{1}{8}$ acres of land. If he sells $2\frac{1}{4}$ acres to his friend Eric Hand, how many acres does he have left?

[1–2]

Perform the indicated operations on decimal numbers.

20. 20.6 + 1.373 + 210.42 + 0.027 + 31.09

21. 42.5 − 10.705

22. 213.4 × 6.35

23. 316.03 ÷ 22.1

24. Peter purchased an automobile for $3,450.63 and sold it for $4,016.12. How much profit did Peter make on the sale?

25. Linda owns 3 pieces of property 2.34, 3.61, and 1.91 acres in size. How many total acres of property does she own?

26. An automobile uses 15.2 gallons of gasoline to travel 188.8 miles. How many miles per gallon did the automobile average?

Find the following percentages.

27. 4% of 250

28. 57% of 120

29. 62.5% of 40

30. 131.2% of 60

[1–3]

List the elements of the following sets.

31. Integers between 49 and 56

32. Natural numbers less than 5

33. Whole numbers that are not natural numbers

34. Integers between −4 and 4

Plot the graphs of the following numbers, using a different number line for each problem.

35. $-2, -\frac{1}{2}, 0, 3$

36. $-\frac{3}{4}, 1, \frac{3}{2}, \frac{5}{2}$

37. $-4, -1, \sqrt{2}, 4$

38. $-3, -2, \frac{1}{2}, \pi$

Replace the ? with the proper inequality symbol ($<$ or $>$) to get a true statement.

39. 4 ? 8

40. −5 ? 0

41. −10 ? −20

42. $|-10|$? $|-20|$

43. $|-5|$? $|0|$

44. $|-8|$? $|4|$

[1–4, 1–5]

Find the sum or difference.

45. (−1) + (−3)

46. 6 − 3

47. 7 − 13

48. (−4) + (5)

49. (−8) + (2)

50. 7 − (−8)

51. (−8) − (−4)

52. (−3) − (6)

53. 0 + (−3) − (−7) + 3 − (+4)

54. 4 − 3 + 7 − 8 + 12 − (−3)

[1–6, 1–7]

Find the product or quotient. If a quotient does not exist, so state.

55. $3 \cdot (-7)$

56. $(-4) \cdot (-3)$

57. $(-8) \cdot (3) \cdot (-1)$

58. $8 \cdot (-9) \cdot (-1) \cdot (-2)$

59. $(-4) \cdot (3) \cdot (-5) \cdot 0$

60. $\dfrac{-14}{2}$

61. $\dfrac{-8}{-4}$

62. $24 \div (-4)$

63. $\dfrac{7}{0}$

64. $\dfrac{0}{-8}$

65. $\dfrac{0}{0}$

66. $\dfrac{(-2)(-3)}{-6}$

67. Find two integer factors of 36 whose sum is -13.

68. A man suffers successive financial losses of \$3,000, \$2,560, and \$3,300 on three business transactions. A loss is denoted by a negative number, and the man originally had \$52,000.

a. Write a statement using negative numbers representing his assets after the losses.

b. Find the total assets after the losses.

69. At 7 A.M. the temperature was $-17°$. At noon that same day the reading was 23°. How much of a rise in temperature was there from 7 A.M. to noon?

70. The temperature readings during a five-hour period were 63°, 72°, 80°, 75°, and 69°.

a. Represent by positive and negative integers how much rise (+) and fall (−) there was from hour to hour.

b. Was the numerical value of the total rise greater than, equal to, or less than that of the total fall? How much? (Represent by a positive or negative integer.)

c. If the sixth hour showed a drop of 11°, what was the temperature during the sixth hour? Write a statement involving a negative integer representing this answer.

[1–8]

Perform the indicated operations and simplify.

71. $(-5)^2$

72. -4^3

73. -4^2

74. -3^3

75. $100 - 4 \cdot 5 + 18$

76. $-7 + 14 \div 7 + 2$

77. $18 + 3 \cdot 12 \div 2^2 - 7$

78. $19 - (14 - 6) + 7^2 - 11$

79. $\dfrac{8(2-4)}{4} - \dfrac{35}{7}$

80. $4[8 - 2(5 - 3) + 1]$

81. $\left[\dfrac{8 + (-2)}{-3}\right]\left[\dfrac{14 \div (-2)}{-1}\right]$

82. $\left[\dfrac{(-12) + (-6)}{4}\right]\left[\dfrac{(-18)(-3)}{-9}\right]$

APPENDIX

Answers and Solutions

Chapter 1

Exercise 1–1

Answers to odd-numbered problems

1. $\frac{1}{2}$ **3.** $\frac{5}{6}$ **5.** $\frac{8}{9}$ **7.** $\frac{7}{9}$ **9.** 2 **11.** $\frac{17}{20}$ **13.** $\frac{1}{2}$
15. $\frac{49}{96}$ **17.** $\frac{7}{12}$ **19.** $\frac{15}{28}$ **21.** $\frac{2}{7}$ **23.** $\frac{25}{17}$ **25.** $\frac{51}{7}$ or $7\frac{2}{7}$
27. $\frac{132}{7}$ or $18\frac{6}{7}$ **29.** 12 **31.** 12 **33.** $\frac{21}{32}$ **35.** $\frac{1}{5}$
37. $\frac{32}{21}$ or $1\frac{11}{21}$ **39.** $\frac{20}{7}$ or $2\frac{6}{7}$ **41. a.** $187\frac{17}{48}$ in.^{3} **b.** $31\frac{39}{64}$ in.^{3}
43. 120 **45.** 126 **47.** 144 **49.** 385 **51.** 120 **53.** 60
55. $\frac{2}{3}$ **57.** $\frac{7}{12}$ **59.** $\frac{3}{5}$ **61.** $\frac{13}{8}$ or $1\frac{5}{8}$ **63.** $\frac{17}{5}$ or $3\frac{2}{5}$
65. $\frac{16}{15}$ or $1\frac{1}{15}$ **67.** $\frac{7}{24}$ **69.** $\frac{149}{270}$ **71.** $\frac{11}{20}$ **73.** $\frac{57}{16}$ or $3\frac{9}{16}$
75. $\frac{5}{3}$ or $1\frac{2}{3}$ **77.** $86\frac{1}{2}$ ft **79.** $10\frac{11}{12}$ lb

Solutions to trial exercise problems

7. $\frac{28}{36} = \frac{4\cdot 7}{4\cdot 9} = \frac{7}{9}$ **21.** $\frac{6}{7} \div 3 = \frac{6}{7}\cdot\frac{1}{3} = \frac{6\cdot 1}{7\cdot 3} = \frac{2\cdot 3}{7\cdot 3} = \frac{2}{7}$

27. $7\frac{1}{3}\cdot 2\frac{4}{7} = \frac{22}{3}\cdot\frac{18}{7} = \frac{22\cdot 3\cdot 3\cdot 2}{3\cdot 7} = \frac{22\cdot 3\cdot 2}{7} = \frac{132}{7}$ or $18\frac{6}{7}$

33. $\dfrac{\frac{7}{8}}{\frac{4}{3}} = \frac{7}{8} \div \frac{4}{3} = \frac{7}{8}\cdot\frac{3}{4} = \frac{7\cdot 3}{8\cdot 4} = \frac{21}{32}$

35. $\frac{4}{5}\cdot\frac{2}{3}\cdot\frac{3}{8} = \frac{4\cdot 2\cdot 3}{5\cdot 3\cdot 8} = \frac{1\cdot(4\cdot 2\cdot 3)}{5\cdot(4\cdot 2\cdot 3)} = \frac{1}{5}$

42. $61\frac{1}{2} \div 14 = \frac{123}{2}\cdot\frac{1}{14} = \frac{123}{28} = 4\frac{11}{28}$ in.

45. $6 = 2\cdot 3$
$14 = 2\cdot 7$ LCD is $2\cdot 3\cdot 3\cdot 7 = 126$.
$18 = 2\cdot 3\cdot 3$

61. $1 + \frac{5}{8} = \frac{8}{8} + \frac{5}{8} = \frac{8+5}{8} = \frac{13}{8}$ or $1\frac{5}{8}$

71. $\frac{7}{15} + \frac{5}{6} - \frac{3}{4} = \frac{7}{15}\cdot\frac{4}{4} + \frac{5}{6}\cdot\frac{10}{10} - \frac{3}{4}\cdot\frac{15}{15} = \frac{28}{60} + \frac{50}{60} - \frac{45}{60}$
(LCD is 60) $= \frac{28+50-45}{60} = \frac{33}{60} = \frac{11}{20}$

77. $P = 2\ell + 2w = 2\cdot 24\frac{1}{2} + 2\cdot 18\frac{3}{4} = 2\cdot\frac{49}{2} + 2\cdot\frac{75}{4}$
$= 49 + \frac{75}{2} = \frac{98}{2} + \frac{75}{2} = \frac{98+75}{2} = \frac{173}{2}$
$= 86\frac{1}{2}$ ft

Exercise 1–2

Answers to odd-numbered problems

1. $\frac{2}{5}$ **3.** $\frac{3}{20}$ **5.** $\frac{1}{8}$ **7.** $\frac{7}{8}$ **9.** 19.019 **11.** 540.2927
13. 13.5585 **15.** 156.9876 **17.** 1.06964 **19.** 9.52816
21. 0.428412 **23.** 0.9100081809 **25.** 1.2 **27.** 40 **29.** 102
31. 2,500 **33.** 0.15 **35.** 0.65 **37.** $0.\overline{2}$ **39.** $12.91
41. 13 cardinals **43.** 0.57 sec **45.** 122.28 gal
47. 1097.222 yd^2 **49.** $0.05 = \frac{1}{20}$ **51.** $0.12 = \frac{3}{25}$
53. $1.35 = \frac{27}{20}$ or $1\frac{7}{20}$ **55.** $3.25 = 3\frac{1}{4}$ or $\frac{13}{4}$ **57.** $\frac{4}{5}$, 80%
59. $\frac{27}{50}$, 54% **61.** $\frac{23}{20}$, 115% **63.** 0.75, 75% **65.** 0.375, 37.5%
67. 2 **69.** 33.8 **71.** 550 **73.** $256.50 **75.** $23
77. $8.50 discount, $25.50 discount price **79.** 0.96 oz

Solutions to trial exercise problems

3. $0.15 = \frac{15}{100} = \frac{3\cdot 5}{20\cdot 5} = \frac{3}{20}$

13. $10.03 + 3.113 + 0.3342 + 0.0763 + 0.005 = $

$$\begin{array}{r} 10.0300 \\ 3.1130 \\ 0.3342 \\ 0.0763 \\ \underline{0.0050} \\ 13.5585 \end{array}$$

19. $(7.006)(1.36) = $

$$\begin{array}{r} 7.006 \\ \underline{1.36} \\ 42036 \\ 21018 \\ \underline{7006} \\ 9.52816 \end{array}$$

28. $21.681 \div 8.03 = 2.7$

$$\begin{array}{r} 2.7 \\ 8.03\overline{)21.681} \\ \underline{16\ 06} \\ 5\ 621 \\ \underline{5\ 621} \\ 0 \end{array}$$

33. $\frac{3}{20} = 20\overline{)3.000} = 0.15$ (quotient .15)
20
100
100

37. $\frac{2}{9} = 9\overline{)2.000} = 0.\bar{2}$ (repeating) (quotient .222...)
18
20
18
20

39. 14.36
0.899 (89.9¢ = \$0.899)
12924
12924
11488
12.90964 ≈ \$12.91

51. $12\% = 12.\% = 0.12$
$12\% = \frac{12}{100} = \frac{3 \cdot 4}{25 \cdot 4} = \frac{3}{25}$

59. $0.54 = 0.54. = 54\%$
$= \frac{54}{100} = \frac{27}{50}$

62. $2.40 = 2\frac{2}{5} = \frac{12}{5} = 240\%$

69. 26% of 130 = 0.26 × 130 = 130
0.26
780
260
33.80 = 33.8

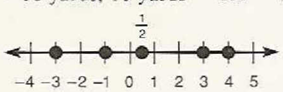
77. discount = 25% of 34
= 0.25 × 34 = \$8.50
price = 34.00 − 8.50 = \$25.50

Exercise 1–3

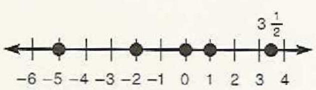
Answers to odd-numbered problems

1. {Sunday,Monday,Tuesday,Wednesday,Thursday,Friday,Saturday} **3.** {January,February,March} **5.** {January,March,May,July, August,October,December} **7.** {a,l,g,e,b,r} **9.** {i,n,t,e,r,m,d,a} **11.** {3,5,7,9} **13.** {Sunday,Saturday} **15.** −\$10, \$150 **17.** −10 yards, 16 yards **19.** −14 points, 8 points

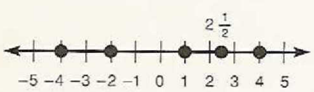
21.



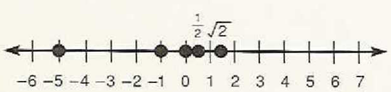
23.



25.



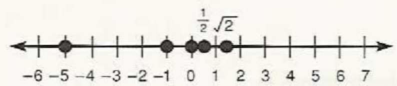
27.



29. $-6,-4,0,3,6$ **31.** $-11,-7,-4,-1,2$ **33.** $-2,3,4,5,11$ **35.** $-1\frac{3}{4},-\frac{1}{2},1\frac{1}{2},3,4$ **37.** $4 < 8$ **39.** $9 > 2$ **41.** $-3 > -8$ **43.** $-10 < -5$ **45.** $0 < 4$ **47.** $0 > -6$ **49.** 2 **51.** 5 **53.** 4 **55.** $\frac{1}{2}$ **57.** $1\frac{1}{2}$ **59.** $-\frac{5}{8}$ **61.** -6 **63.** $|5|<|-7|$ **65.** $|0|<|-2|$ **67.** $|-8|>|-5|$ **69.** $|-6|>|-2|$ **71.** $7>|-2|$ **73.** $|-4|< 6$ **75.** $|-27|$ **77.** $|-9|$

Solutions to trial exercise problems

27.



For the location of $\sqrt{2}$, we use the approximation 1.414 from a calculator.
35. $-1\frac{3}{4},-\frac{1}{2},1\frac{1}{2},3,4$ The values represent an approximation of the coordinates. **40.** $-2 > -4$, since -2 lies to the right of -4 on the number line. **46.** $-3 < 0$, since -3 lies to the left of 0 on the number line.

Exercise 1–4

Answers to odd-numbered problems

1. −13 **3.** 5 **5.** −5 **7.** −5 **9.** 0 **11.** −13.6 **13.** −11.1 **15.** $-\frac{1}{2}$ **17.** $\frac{1}{10}$ **19.** $-\frac{3}{4}$ **21.** 3 **23.** 10 **25.** −44 **27.** −22 **29.** −10 **31.** 0 **33.** 11 **35.** 15 **37.** 7° *C* **39.** \$32 **41.** 2 mb **43.** 99 **45.** −41° *F* **47.** \$64

Solutions to trial exercise problems

11. $(-8.7) + (-4.9) = -13.6$ The signs are the same so we add their absolute values $8.7 + 4.9 = 13.6$ and prefix this sum by their common sign. **15.** $\left(-\frac{1}{6}\right) + \left(-\frac{1}{3}\right) = -\frac{1}{2}$ The signs are the same so we add their absolute values $\frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{1+2}{6}$ $= \frac{3}{6} = \frac{1}{2}$ and prefix this sum by their common sign.
21. $10 + (-5) + (-2) = 5 + (-2) = 3$ The numbers were added left to right. **23.** $(-12) + (-10) + (+8) + (+24)$ $= (-22) + (+8) + (+24) = (-14) + (+24) = 10$
34. The sum of 15 and −18 increased by 10
15 + (−18) + 10
15 + (−18) + 10 = −3 + 10 = 7
44. Let t = the temperature at 1 P.M. To find the new temperature, we must *add* the rise in temperature to the original temperature.
temperature at 1 P.M. is temperature at 8 A.M. rose 39°
$t = -13 + 39$
$t = -13 + 39$
$t = 26$
The temperature at 1 P.M. was 26° *F*.

Exercise 1–5

Answers to odd-numbered problems

1. -1 **3.** 6 **5.** -12 **7.** -5 **9.** -4 **11.** 14 **13.** -6 **15.** 7 **17.** $-\frac{1}{4}$ **19.** $2\frac{5}{8}$ **21.** -9.4 **23.** -312 **25.** 301.8 **27.** -24 **29.** 11 **31.** -53 **33.** 16 **35.** -3 **37.** 10 **39.** -8 **41.** -7 **43.** $-38°\ C$ **45.** $-\$372$ **47.** 6 feet **49.** -22 **51.** -9 **53.** 60 **55.** 19 **57.** -13 **59.** -5 **61.** \$8 **63.** 38° **65.** 14 years old **67.** \$245 **69.** 20,602 feet **71.** \$19

Solutions to trial exercise problems

13. $(-6) + 0 = -6$ The sum of zero and a number is that number. **15.** $7 - 0 = 7$ A number minus zero is that number. **26.** $(-12) - (-10) - (8) = (-12) + (10) + (-8)$ $= (-2) + (-8) = -10$ **38.** $12 + 3 - 16 - 10 - (12 + 5)$ $= 12 + 3 - 16 - 10 - (17) = 15 - 16 - 10 - (17)$ $= -1 - 10 - (17) = -11 - (17) = -28$
65. Let a = the age that Erin will be in the year 2000. We must find the *difference* between 2000 and 1986.

age in the year 2000	is	the difference between		
		2000	and	1986
a	=	2000	−	1986

$a = 2000 - 1986$
$a = 14$
Erin will be 14 years old in the year 2000.

Exercise 1–6

Answers to odd-numbered problems

1. 15 **3.** -28 **5.** -60 **7.** -36 **9.** 20 **11.** -105 **13.** -4.32 **15.** -13.769 **17.** -2.16 **19.** $\frac{9}{16}$ **21.** $\frac{1}{4}$ **23.** -120 **25.** 144 **27.** 0 **29.** 0 **31.** $-5,6$ **33.** $-5,-4$ **35.** $-7,0$ **37.** $3,-4$ **39.** $7,5$ **41.** $8,-1$ **43.** $4,-3$ **45.** $6,-3$ **47.** \$76 **49.** 1,050 people **51.** \$975 **53.** $-\$92$ **55.** 525¢ = \$5.25 **57.** 700 gallons

Solutions to trial exercise problems

11. $7 \cdot (-1)(-3)(-5) = (-7)(-3)(-5) = (21)(-5) = -105$; negative answer because there were an odd number of negative factors **27.** $(-2)(0)(3)(-4) = 0$ When zero is one of the factors, zero will be the answer. **30.** $-16, 0$ Since $(-4)(4) = (-16)$ and $(-4) + (4) = 0$, then -4 and 4 are the integers. **31.** $-30, 1$ Since $(-5)(6) = (-30)$ and $(-5) + (6) = (1)$, then -5 and 6 are the integers. **48.** Assets $(5)(-6)$, and his assets would change by (-30) dollars. **57.** Let g = the number of gallons of milk sold in 4 weeks. Since there are 28 days in 4 weeks, we must *multiply* 28 by 25 to determine the gallons of milk sold.

total gallons of milk sold	is	28 days	at	25 gallons per day
g	=	28	·	25

$g = 28 \cdot 25$
$g = 700$
The grocer sold 700 gallons of milk.

Exercise 1–7

Answers to odd-numbered problems

1. 2 **3.** -8 **5.** 2 **7.** -8 **9.** undefined **11.** 0 **13.** indeterminate **15.** -7 **17.** -5 **19.** -2 **21.** -4 **23.** 0 **25.** -3 **27.** undefined **29.** indeterminate **31.** $-1°\ C$ **33.** 6 hours **35.** 9 seconds **37.** \$4 **39.** 25 miles **41.** 4° **43.** 32 books **45.** 12 minutes

Solutions to trial exercise problems

11. $\frac{0}{-9} = 0$, since $(-9) \cdot 0 = 0$ **19.** $\frac{(-4)(-3)}{-6} = \frac{12}{-6}$ $= -2$; odd number of negative factors **22.** $\frac{(-4)(0)}{-8} = \frac{0}{-8}$ $= 0$, since $(-8) \cdot 0 = 0$ **26.** $\frac{(-2)(-4)}{(0)(4)} = \frac{8}{0}$ is undefined **29.** $\frac{(-6)(0)}{(-3)(0)} = \frac{0}{0}$ = indeterminate **33.** number of hours $= \dfrac{\text{number of miles}}{\text{rate of travel in miles per hour}}$. Hence $\frac{282}{47} = 6$; 6 hours.
45. Let m = the number of minutes it took Alice to run 1 mile. Since there are 60 minutes in 1 hour, the race took 300 minutes + 12 minutes, which is 312 minutes. We must divide 312 minutes by 26 miles to determine the number of minutes per mile.

minutes per mile	is	number of minutes	divided by	number of miles
m	=	312	÷	26

$m = 312 \div 26 = 12$
Alice ran 1 mile in 12 minutes.

Exercise 1–8

Answers to odd-numbered problems

1. 16 **3.** -27 **5.** -36 **7.** -1 **9.** 1 **11.** 4 **13.** 26 **15.** 0 **17.** 3 **19.** 6 **21.** $\frac{12}{7}$ or $1\frac{5}{7}$ **23.** 11 **25.** 46 **27.** 2 **29.** 121 **31.** $\frac{5}{24}$ **33.** 96 **35.** -48 **37.** 78 **39.** 3 **41.** 38 **43.** 4 **45.** 12 **47.** -15.99 **49.** 38.47 **51.** 29 **53.** 33 **55.** $\frac{19}{288}$ **57.** $23\frac{1}{3}°\ C$ **59.** $\frac{110}{7}$ or $15\frac{5}{7}$ square inches **61.** $\frac{288}{41}$ or $7\frac{1}{41}$ inches **63.** \$374.50 **65.** 8 **67.** 2,728

Solutions to trial exercise problems

2. $(-5)^4 = (-5)(-5)(-5)(-5) = (25)(-5)(-5)$ $= (-125)(-5) = 625$; positive since we have a negative number to an even power **3.** $(-3)^3 = (-3)(-3)(-3) = (9)(-3)$ $= -27$; negative since we have a negative number to an odd power **8.** $-2^2 = -(2 \cdot 2) = -(4) = -4$ **17.** $0(5 + 2) + 3$ $= 0(7) + 3 = 0 + 3 = 3$ **41.** $4(2 - 5)^2 - 2(3 - 4)$ $= 4(-3)^2 - 2(3 - 4) = 4(-3)^2 - 2(-1) = 4(9) - 2(-1)$ $= 36 - 2(-1) = 36 - (-2) = 38$ **43.** $\frac{5(3-5)}{2} - \frac{27}{-3}$ $= \frac{5(-2)}{2} - \frac{27}{-3} = \frac{-10}{2} - \frac{27}{-3} = (-5) - \frac{27}{-3} = (-5) - (-9)$ $= 4$ **50.** $5[10 - 2(4 - 3) + 1] = 5[10 - 2(1) + 1]$ $= 5[10 - 2 + 1] = 5[8 + 1] = 5[9] = 45$

54. $\left(\frac{6-3}{7-4}\right)\left(\frac{14+2\cdot 3}{5}\right)=\left(\frac{3}{3}\right)\left(\frac{14+6}{5}\right)=\left[\frac{3}{3}\right]\left[\frac{20}{5}\right]$
$=1\cdot 4=4$ **59.** $\frac{22}{7}\cdot 3^2-\frac{22}{7}\cdot 2^2=\frac{22}{7}\cdot 9-\frac{22}{7}\cdot 4$
$=\frac{198}{7}-\frac{88}{7}=\frac{198-88}{7}=\frac{110}{7}$ or $15\frac{5}{7}$ square inches

65. Let $p=$ the total number of pieces of lumber. *Dividing* the 16-foot board by 4 and the 12-foot board by 3 will give us the number of pieces of lumber. If we *add* the number of pieces from the 16-foot board to the number of pieces from the 12-foot board, we will have the total number of pieces.

total number of pieces	is	number of pieces from the 16-foot board	combined with	number of pieces from the 12-foot board
p	$=$	$16\div 4$	$+$	$12\div 3$

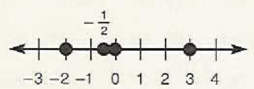
$p=16\div 4+12\div 3$
$=4+4$ Priority 3
$=8$ Priority 4

There will be 8 pieces of lumber.

Chapter 1 review

1. $\frac{5}{7}$ **2.** $\frac{3}{4}$ **3.** $\frac{2}{3}$ **4.** $\frac{10}{7}$ or $1\frac{3}{7}$ **5.** $\frac{3}{5}$ **6.** $\frac{21}{20}$ or $1\frac{1}{20}$
7. $\frac{7}{8}$ **8.** $\frac{25}{8}$ or $3\frac{1}{8}$ **9.** $\frac{25}{3}$ or $8\frac{1}{3}$ **10.** $\frac{5}{8}$ acre **11.** $\frac{2}{5}$ cup
12. $\frac{8}{7}$ or $1\frac{1}{7}$ **13.** $\frac{19}{24}$ **14.** $\frac{5}{6}$ **15.** $\frac{2}{9}$ **16.** $6\frac{17}{20}$ **17.** $\frac{2}{15}$
18. $\frac{7}{12}$ **19.** $\frac{7}{8}$ acre **20.** 263.51 **21.** 31.795 **22.** 1,355.09
23. 14.3 **24.** \$565.49 **25.** 7.86 acres **26.** $\approx$ 12.42 mpg
27. 10 **28.** 68.4 **29.** 25 **30.** 78.72 **31.** $\{50,51,52,53,54,55\}$
32. $\{1,2,3,4\}$ **33.** $\{0\}$ **34.** $\{-3,-2,-1,0,1,2,3\}$

35.



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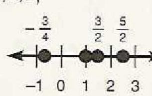

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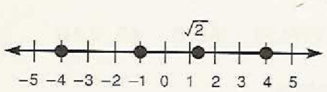

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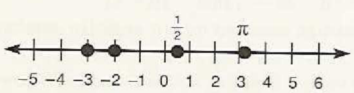

39. $<$ **40.** $<$ **41.** $>$ **42.** $<$ **43.** $>$ **44.** $>$ **45.** -4
46. 3 **47.** -6 **48.** 1 **49.** -6 **50.** 15 **51.** -4
52. -9 **53.** 3 **54.** 15 **55.** -21 **56.** 12 **57.** 24
58. -144 **59.** 0 **60.** -7 **61.** 2 **62.** -6 **63.** undefined
64. 0 **65.** indeterminate **66.** -1 **67.** $-9,-4$
68. **a.** $52{,}000+(-3{,}000)+(-2{,}560)+(-3{,}300)$ **b.** \$43,140
69. 40° **70.** **a.** $+9,+8,-5,-6$ **b.** $>,+6$ **c.** 58°, $69+(-11)=58$ **71.** 25 **72.** -64 **73.** -16 **74.** -27
75. 98 **76.** -3 **77.** 20 **78.** 49 **79.** -9 **80.** 20
81. -14 **82.** 27

Chapter 2

Exercise 2–1

Answers to odd-numbered problems

1. 2 terms **3.** 3 terms **5.** 1 term **7.** 3 terms **9.** 2 terms **11.** 1 term **13.** 2 terms **15.** 5 is the coefficient of x^2, 1 is understood to be the coefficient of x, -4 is the coefficient of z **17.** 1 is understood to be the coefficient of x, -1 is understood to be the coefficient of y, -3 is the coefficient of z **19.** -2 is the coefficient of a, -1 is understood to be the coefficient of b, 1 is understood to be the coefficient of c **21.** polynomial, binomial **23.** not a polynomial because a variable is in the denominator **25.** not a polynomial because a variable is in the denominator **27.** polynomial, trinomial **29.** $b-3a$ **31.** $y+5$ **33.** $x(y+z)$ **35.** $a-b$ **37.** (let $x=$ the number) $x-12$ **39.** (let $x=$ the number) $3x+1$ **41.** (let $x=$ the number) $2(x+4)$

Solutions to trial exercise problems

8. $\frac{15x^2+y}{8}$ has one term because the fraction bar is a grouping symbol. **24.** $\frac{a+b}{5}-c$ is a polynomial. A constant can appear in the denominator, a variable cannot. **36.** $\frac{1}{2}$ of x, decreased by 2 times x would be $\frac{1}{2}x-2x$. **39.** 3 times a number, increased by 1: If we let x represent the number, then we would have $3x+1$.

Review exercises

1. -25 **2.** 64 **3.** -2 **4.** 15 **5.** 22 **6.** 23

Exercise 2–2

Answers to odd-numbered problems

1. 9 **3.** 5 **5.** 48 **7.** 5 **9.** 62 **11.** 288 **13.** 61 **15.** 0 **17.** -1 **19.** 1 **21.** -44 **23.** 20 **25.** 0 **27.** 43 **29.** $\frac{20}{3}$ **31.** 160 **33.** 288 **35.** 54 **37.** 2,140 **39.** 114 **41.** 256 **43.** 6 **45.** $\frac{15{,}000}{857}$ **47.** $\frac{540}{13}$ **49.** $\frac{400}{33}$ **51.** $85m$ **53.** $\frac{y}{10}$ **55.** $5n+10d$ **57.** **a.** $p+12$ **b.** $p-5$ **59.** $258-n+m$ **61.** $\frac{c}{50}$ **63.** $y+2$ **65.** $12f+t$ **67.** $25{,}000n-2{,}000$ **69.** $(9.95)p+(12.99)q$

Solutions to trial exercise problems

5. $(3a+2b)(a-c)=[3(\ \ )+2(\ \ )][(\ \ )-(\ \ )]$
$=[3(2)+2(3)][(2)-(-2)]=[6+6][4]=[12][4]=48$
14. $(4a+b)-(3a-b)(c+2d)=[4(\ \ )+(\ \ )]$
$-[3(\ \ )-(\ \ )][(\ \ )+2(\ \ )]=[4(2)+(3)]$
$-[3(2)-(3)][(-2)+2(-3)]=[8+3]$
$-[6-3][(-2)+(-6)]=[11]-[3][-8]=[11]-[-24]$
$=35$ **31.** $I=prt;\ I=(\ \ )(\ \ )(\ \ )=(1{,}000)(0.08)(2)=(80)(2)$
$=160$ **39.** $A=\frac{I^2R-120E^2}{R};\ A=\frac{(\ \ )^2(\ \ )-120(\ \ )^2}{(\ \ )}$
$=\frac{(12)^2(100)-120(5)^2}{(100)}=\frac{(144)(100)-120(25)}{100}$
$=\frac{14{,}400-3{,}000}{100}=\frac{11{,}400}{100}=114$ **47.** $V=\frac{vn}{N};\ V=\frac{(\ \ )(\ \ )}{(\ \ )}$

Contents

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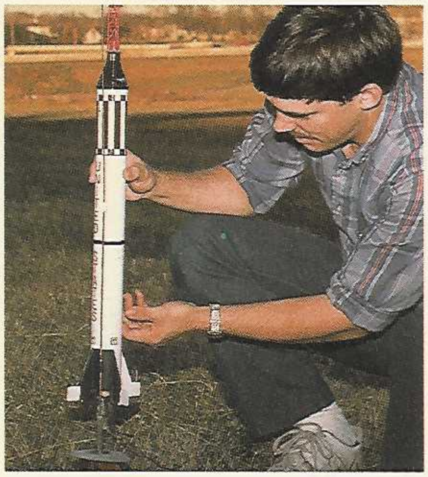
Chapter 4 ■ Factoring and solution of quadratic equations by factoring



Chapter 5 ■ Rational Expressions, Ratio and Proportion

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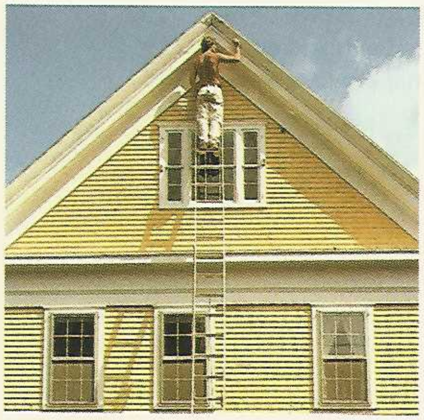
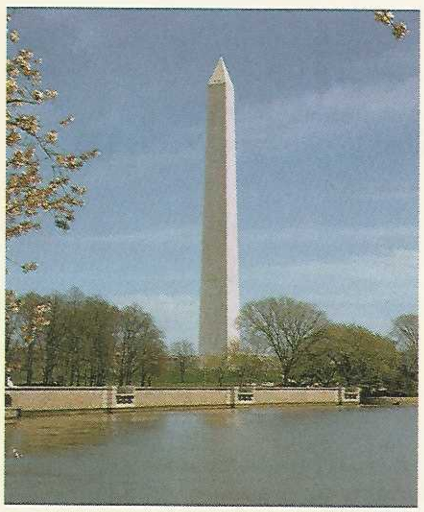


Chapter 10 ■ Solutions of Quadratic Equations



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